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Inventory, investment, and pricing policies for lot-size decision makers

Toshitsugu Otake
Iowa State University

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Inventory, investment, and pricing policies for lot-size decision makers

by

Toshitsugu Otake

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300 North Zeeb Road
Ann Arbor, MI 48103

Graduate College
Iowa State University

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Major Professor

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For the Major Program

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GENERAL INTRODUCTION

General Background and Objectives

In this dissertation, inventory, investment, and pricing policies for lot-size decision makers are examined based on classical economic order quantity (EOQ; see e.g., Hillier and Lieberman 1995). Specifically, we focus on investment in setup operations, investment in quality improvement, and market dependent products such as substitutes and complements. We examine various impacts of investment and competition on inventory policies and derive managerial insights and economic implications. Throughout this dissertation, deterministic mathematical programming is used as the primary analysis technique and optimal policies are obtained through this technique.

The primary objectives and contributions of this dissertation are as follows: Our objectives consist of examination of 1) inventory and investment relationships as well as 2) inventory and competition relationships.

For the inventory and investment relationships, we construct and analyze inventory and investment in setup operations policies, inventory and investment in quality improvement policies, and inventory and capital investment allocation policies in setup

and quality operations under return on investment (ROI) maximization, where ROI is defined as the ratio of profit to average investment. In each analysis, we consider the benchmark problem, in which a decision maker does not have an option to invest any additional money. Based on this problem, we consider the another problem, in which a decision maker has an option to invest additional money in setup operations and/or quality improvement. The resulting contributions are the establishment of an ROI model with/without the capital budget constraints and characterization of the unique global optimal solution when there exists an option to invest in setup operations. We also show how the inventory level is reduced when it is optimal to invest additional money in setup operations and/or quality improvement. Furthermore, we are comparing and contrasting inventory and investment policies under ROI maximization with those policies under other economic/finance performance criteria such as cost minimization and profit maximization.

For the inventory and competition relationships, on the other hand, we design and analyze two duopoly (two sellers) models for two profit maximizing sellers when products are substitutes or complements. Competition is characterized by the Cournot-type model, in which each firm predicts the other firm's quantity first in deciding its own quantity, and the Bertrand-type model, in which Each firm predicts the other firm's price first in deciding its own price. The resulting contributions are formulation of inventory and pricing policies for substitutes and complements. Furthermore, we obtain the closed-form inventory and pricing policies at equilibrium when symmetric demand and cost are assumed.

Now, we focus on overall background for this dissertation. Traditionally, there are numerous papers analyzing cost minimization and profit maximization, and ROI maximization as economic/finance performance criteria (see e.g., Schroeder and Krishnan 1976). Furthermore, there are numerous papers analyzing setup cost reduction inventory models, quality improvement inventory models and capital allocation inventory models (see e.g., Porteus 1985; Lee and Rosenblatt 1985; Hong, Xu, and Hayya 1993). Given price and demand rate, however, in deciding the optimal level of investment for quality improvement and setup operations, it would be inherently suboptimal for ROI maximizing decision makers to utilize the existing models constructed for cost-minimizing/profit-maximizing decision makers. This dissertation is motivated by the lack of mathematical models with ROI as an economic/finance performance criterion when the option of investing in quality improvement and/or setup operations exists.

The just-in-time (JIT) or zero inventory philosophy leads to reduction in the lot size as small as possible. Investing in setup operations is an important aspect of the JIT philosophy. However, when a production process is not reliable, the JIT philosophy is not efficient, e.g., loss of sales. Hence, it is also important to consider investment in quality improvement to apply for the JIT philosophy (see e.g., Voss 1987).

Also, in this dissertation, competition is characterized by duopoly (two sellers), a Cournot-type model and a Bertrand-type model (see e.g., Varian 1992). Furthermore, characterization of products used for duopoly models in this dissertation is considered as substitutes and complements. Substitutes are products that can be substituted for each other such as coffee and tea. On the other hand, complements are products that can be

used together such as coffee and sugar. Each seller is assumed to produce one product, and his competitor is assumed to produce substitutes or complements. Even though there have been extensive studies of substitutes and complements in the literature of economic theory, to my knowledge, there have been few papers dealing with substitutes and complements in the context of inventory policies. Hence, given the prevalence of substitutes and complements in the real world, it is highly desirable to derive economic implications and managerial insights in the context of inventory. In thses analyses, profit maximization is used as economic/finance performance criterion.

Finally, we note that the classical EOQ model has been studied continuously for several decades and numerous extensions have been made (see e.g., Arcelus and Rowcroft 1992). Furthermore, various papers in the industrial engineering literature have utilized the EOQ type models or measured their own models against EOQ type models (e.g., Liao and Shyu 1991; Johnson and Montgomery 1994). Likewise, in this dissertation, we attempt to preserve the general framework of the EOQ model as much as possible, while extending it to the case of inventory, investment, and pricing policies.

Thus far, we have discussed overall objectives, key contributions, and background. Let us now proceed to discuss the overall scope of this dissertation. First of all, all variables and parameters are assumed to be deterministic. Under a traditional EOQ model, the demand rate of product is considered to be constant and deterministic because the lifetime of product is assumed at maturity for the product life cycle, development – growth – shakeout – maturity – saturation (see e.g., Nahmias 1989). As a dependent demand system, in an integrated manufacture system including the wholesale and retail

level, the production-lot-sizing decisions at one level of the system result in the demand patterns at other levels. The interaction between them plays an important role and this is called materials requirements planning systems (MRP; see e.g., Nahmias 1989). In this dissertation, we will not focus on dependent demand system such as MRP.

Now, let us examine cost components in this dissertation. Under the profit maximization model (see e.g., Whitin 1955; Ladany and Sternlieb 1974), there are three components of the total cost: the holding cost, the setup cost, and the unit variable cost. The holding cost is the cost of warehousing, taxes and insurance, damaging or losing, and any other cost directly related to the amount of inventory on hand. The setup cost is the fixed cost independent of the size of the order such as machine changeovers, postage, and telephone calls. The unit variable cost is the cost depending on the amount of inventory procured. Under the ROI maximization model (see e.g., Schroeder and Krishnan 1976; Rosenberg 1991), in addition to the above three components, we will consider capital investment in setup operations and/or quality improvement.

As for additional assumptions in this dissertation, we follow the traditional inventory assumptions such as the replenishment rate is infinite, no shortage is allowed, and there is no delivery lag unless otherwise specified. If the replenishment rate is considered to be infinite, this is good approximation when the production rate is much larger than the demand rate (see e.g, Nahmias 1989; Banks and Fabrycky 1987). Also, if a shortage occurs, the penalty cost is imposed because there is not sufficient stock on hand to meet a demand. Since we assume the constant and deterministic demand, it is reasonable to assume that there is no shortage. Furthermore, it is very difficult in practice to estimate

the loss-of-goodwill of the penalty cost. Finally, another assumption of the traditional EOQ model is that there is no delivery lag. In order to eliminate variability of the delivery lag, close cooperation with suppliers is required in the traditional EOQ model (see e.g., Nahmias 1989; Silver and Peterson 1985). In addition, it does not consider strikes and weather problems to cause delivery delays periodically in this dissertation.

Additional issues regarding our scope of this dissertation are as follows: Under the inventory control, estimates of future demand by forecasting techniques, such as a moving average forecast and an exponential smoothing forecast, are the initial stage for production scheduling and planning (see Silver and Peterson 1985). Nevertheless, since we consider the constant deterministic demand, we don't further analyze any details in this field. Also, off-setting factors for the inventory control (e.g., discounts and rebates) have been considered as an all-units discount model or an incremental discount model. Under the traditional EOQ model, the unit variable cost is independent of the size of the order so that there is no off-setting factor.

Furthermore, under inflationary economic conditions, traditional inventory models are developed (see e.g., Hariga 1994). However, with current small scale of inflation relative to 1970s and 1980s, both inflation and time value of money are disregarded in this dissertation. Most senior managers view today that keeping inventories does not lead to a measure of wealth but a large potential risk. However, we don't consider risk for firms or sellers through this dissertation. In addition, we note that location theory of sellers has been widely analyzed in the literature of Economics, Finance, and Marketing. However, we do not consider the impact on the location of sellers (e.g., the impact of

distance to the closest site) throughout this dissertation. Also, the costs of two sellers are considered to be the same in a symmetric cost case. This indicates that even though functionally there are the same products, these are different such as color.

Finally, from the above overall scope, we summarize scope and usage of each chapter in Table 1 and Table 2.

Dissertation Organization

Thus far, we have discussed overall backgrounds, objectives, key contributions and scope of this dissertation. Now, we explain the organization of the dissertation. This dissertation consists of six papers published, prepared for submission or submitted in some proceedings and journals.

In Chapter 1, titled "Inventory and Investment in Quality Improvement under Return on Investment Maximization," we construct and analyze inventory and investment in quality improvement policies under return on investment (ROI) maximization. In this paper, a decision maker has an option to invest additional money in quality improvement. We formulate the ROI model and characterize the unique optimal policies consisting of the order quantity and the level of investment in quality improvement. Furthermore, based on no option to invest additional money in quality improvement, we show how inventory is reduced when it is optimal to invest additional money in quality improvement. In addition, we derive closed-form optimal policies and managerial insights when the setup cost is a linear function of the level of investment.

Table 1 Scope of inventory model and usage in each chapter

	Chapter 1	Chapter 2	Chapter 3
Single product	Yes	Yes	Yes
Budget constraints	No	No	Yes
Space constraints	No	No	No
Infinite replenishment rate	Yes	Yes	Yes
Infinite time horizon	Yes	Yes	Yes
Quantity discount	No	No	No
Monopoly	Yes	Yes	Yes
Duopoly	No	No	No
Oligopoly	No	No	No
Perfect competition	No	No	No
Shortage	No	No	No
Lead time	No	No	No
Static demand	Yes	Yes	Yes
Static production rate	Yes	Yes	Yes
Total cost minimization	No	No	No
Profit maximization	No	No	No
ROI maximization	Yes	Yes	Yes
Setup cost reduction	No	Yes	Yes
Perfect product quality	No	Yes	No
Capital investment	Yes	Yes	Yes
Characterization of product	No	No	No

Table 2 Scope of inventory model and usage in each chapter

	Chapter 4	Chapter 5	Chapter 6
Single product	Yes	Yes	Yes
Budget constraints	No	No	No
Space constraints	No	No	No
Infinite replenishment rate	Yes	Yes	Yes
Infinite time horizon	Yes	Yes	Yes
Quantity discount	No	No	No
Monopoly	Yes	No	No
Duopoly	No	Yes	Yes
Oligopoly	No	No	No
Perfect competition	No	No	No
Shortage	No	No	No
Lead time	No	No	No
Static demand	Yes	Yes	Yes
Static production rate	Yes	Yes	Yes
Total cost minimization	No	No	No
Profit maximization	Yes	Yes	Yes
ROI maximization	Yes	No	No
Setup cost reduction	Yes	No	No
Perfect product quality	Yes	Yes	Yes
Capital investment	Yes	No	No
Characterization of product	No	Yes	Yes

In Chapter 2, titled "Inventory and Investment in Setup Operations under Return on Investment Maximization," we construct and analyze inventory and investment in setup operations policies under return on investment (ROI) maximization. We follow the basic model formulation of ROI maximization considered by Chen and Min (see Chen 1995). In this paper, a decision maker has an option to invest additional money in setup operations. We formulate the ROI model and characterize the unique optimal policies consisting of the order quantity and the level of investment in setup operations. Furthermore, based on no option to invest additional money in setup operations, we show how inventory is reduced when it is optimal to invest additional money in setup operations. In addition, we derive closed-form optimal policies and managerial insights when the setup cost is a rational or linear function of the level of investment.

In Chapter 3, titled "Inventory and Capital Investment Allocation Policies under Return on Investment Maximization," we construct and analyze inventory and capital investment allocation policies under return on investment (ROI) maximization. Our model is constructed for a decision maker of a single product with a budget constraint in capital investment. We show how the levels for the prior and posterior order quantities are reduced when it is optimal to invest additional money in setup cost reduction and/or quality improvement. In addition, the unique global optimal solution is determined by employing the primary criterion of ROI maximization, the secondary criterion of the posterior order quantity minimization (i.e., inventory reduction), and the third criterion of the prior order quantity minimization. Moreover, we illustrate a numerical example to show sensitivity analysis of unit variable cost.

In Chapter 4, titled "Inventory and Investment in Setup Operations under Profit and ROI Maximization," we investigate inventory and investment in setup operations policies under a profit maximization model and a return on investment (ROI) maximization model. As in Chapter 2, we follow the basic model formulations of profit/ROI maximization (see Chen's dissertation). Based on these formulations, we examine the corresponding optimality conditions and study how inventory is reduced when it is optimal to invest additional money in setup operations. Furthermore, we compare and contrast the inventory reduction between the profit model and the ROI model.

In Chapter 5, titled "Inventory and Pricing Policies for a Duopoly of Substitute Products," we design and analyze two duopoly models for two profit maximizing sellers. Each seller is assumed to produce one product, and his competitor is assumed to produce a substitute. In characterizing the competitive behavior of each seller, we employ a Cournot-type model and a Bertrand-type model and we derive the equilibrium conditions for both models. Dependency of demand and price are expressed by the linear demand functions, which are widely found in the literature of economics.

In Chapter 6, titled "Inventory and Pricing Policies for a Duopoly of Complements," we design and analyze two duopoly models for two competing sellers. Each seller is assumed to be a profit maximizing EOQ-based decision maker facing linear demand functions. In this paper, based on Cournot-type and Bertrand-type competitive behavioral assumptions, we design and analyze pricing and inventory policies for two sellers. Each seller is assumed to produce one product, and his competitor is assumed to produce a complement. As mentioned before, dependency of demand and price are expressed by

the linear demand functions.

Finally, general concluding remarks in this dissertation are described including chapter reviews and further research followed by references cited in general introduction and general concluding remarks. Overall structure and chapter relationships are summarized in Figure 1.

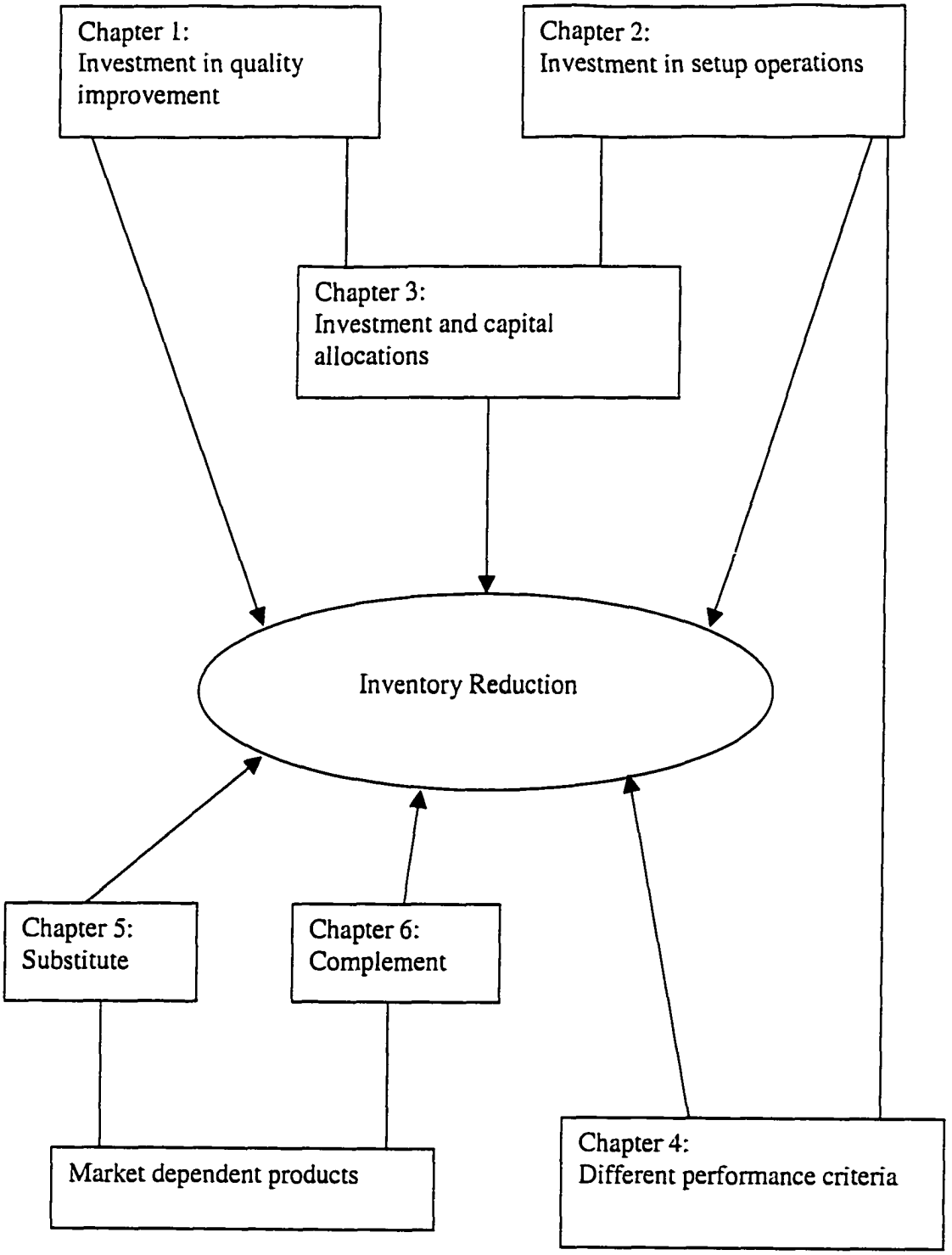

Figure 1 Overall structure and chapter relationships

CHAPTER 1. INVENTORY AND INVESTMENT IN QUALITY IMPROVEMENT UNDER RETURN ON INVESTMENT MAXIMIZATION

A paper submitted to IEEE Transactions on Engineering Management

Toshitsugu Otake and K. Jo Min

Abstract

In this paper, we construct and analyze inventory and investment in quality improvement policies under return on investment (ROI) maximization. In our model, the level of quality is represented by the fraction of an order quantity meeting the quality requirements such as product specifications. The key contributions of this paper are the establishment of an ROI model and characterization of the unique global optimal solution. We also show how the inventory level is reduced when it is optimal to invest additional money in quality improvement. In addition, we derive the unique global optimal solutions in closed-form when the investment in quality improvement is a linear function of the quality. Various interesting managerial insights and a numerical example are provided.

1. Introduction

In this paper, we construct and analyze inventory and investment in quality improvement policies under return on investment (ROI) maximization for a decision maker of an inventory system with a single product. By quality in this paper, we mean the fraction of an order quantity meeting the quality requirements such as product specifications. The primary contributions of this paper are: (1) Formulation of the ROI model and characterization of the unique optimal policies consisting of the levels of order quantity and investment in quality improvement, (2) Characterization of inventory reduction when it is optimal to invest additional money in quality improvement, and (3) Closed-form optimal policies and managerial insights when the investment in quality improvement is a linear function of the quality.

We will now provide the background information for the quality first, the relation between the quality and inventory reduction next, followed by the performance criterion of ROI maximization.

The quality issues for a product in an inventory system have been extensively studied. For example, Lee and Rosenblatt (1985) examine optimal inspection and ordering policies for products with imperfect quality. On the other hand, Cheng (1991) investigates an Economic Production Quantity (EPQ) model with process capability and quality assurance considerations. We note that both papers utilize the fraction of an order quantity that is acceptable (or unacceptable) to indicate the level of quality. Similarly, in our model, we represent the level of quality by the fraction of an order quantity meeting the quality requirements such as product specifications. Hence, the quality im-

provement implies an increase in this fraction. Furthermore, we assume that the quality improvement can be achieved by additional investment in equipment and training.

The relation between the quality and inventory reduction is critical for both practitioners and academics because numerous modern production systems advocate reduction in inventory and improvement in quality. For example, Voss (1987) claims that Just-In-Time production systems lead to increased quality and reduced inventory. In addition, Kekre and Mukhopadhyay (1992) show that there exists a negative relationship between inventory and quality based on empirical results. Moreover, Porteus (1986) studies the process quality improvement and the order quantity in conjunction with setup cost reduction. This work is extended by Hong and Hayya (1995) by considering budget constraints on quality improvement and setup reduction. For the last two papers, we note that the definition of quality is based on a Markovian process model for the probability of the production process becoming out of control, which is a fundamentally different way of looking at the quality of a production system (cf. the definition of quality in Lee and Rosenblatt (1985), Cheng (1991), and this paper).

ROI is a widely utilized economic performance measure dealing with finished goods inventories (see e.g., Schroeder and Krishnan 1976; Morse and Scheiner 1979; and Reece and Cool 1978). Traditionally, numerous papers have employed the profit maximization or cost minimization as their objective in designing and analyzing inventory models (see e.g., Whitin 1955; Smith 1958; Ladany and Sternlieb 1974; Hillier and Lieberman 1995). On the other hand, Schroeder and Krishnan (1976) propose an inventory model under an alternative performance criterion of ROI maximization. Also, Rosenberg (1991)

compares and contrasts profit maximization vs. return on inventory investment with respect to logarithmic concave demand functions.

This paper is motivated by the lack of inventory models under ROI maximization when there exists an option to invest in quality improvement. Since one of the most frequently utilized economic performance criteria in inventory systems other than profit maximization/cost minimization is that of ROI maximization, a comprehensive and quantitative study of ROI maximization is highly desirable. In deciding the optimal level of investment in quality improvement, it would be inherently suboptimal for ROI maximizing decision makers to utilize any other models constructed for profit maximization/cost minimization decision makers. The comprehensive and quantitative study is also desirable because the existing literature qualitatively discusses the link between ROI and the inventory reduction (see e.g., Oakleaf 1972).

The rest of this paper is organized as follows. We first formulate the ROI maximization model for inventory and investment in quality improvement, and characterize the unique global optimal solution. Next, under the assumption of fairly general class of investment function, we show how the inventory level is reduced when it is optimal to invest additional money in quality improvement. Then, for the specific case of a linear investment function, the optimal closed-form solutions are obtained and several interesting managerial insights are presented. Finally, summary and concluding remarks are made.

2. Model Formulation and Optimality Conditions

2.1 Definitions and assumptions

First, various notations and their definitions used in this paper are as follows:

Q: the order quantity size prior to inspection.

r: the fraction of an order quantity meeting the quality requirements.

C: the variable cost per unit including per unit material cost and per unit inspection cost.

I: the inventory holding cost per unit time expressed as a fraction of the unit cost, which excludes the opportunity cost of funds tied up in inventory.

K: the investment in equipment and training so as to increase the level of r.

P: the selling price per unit.

D: the sales quantity per unit time.

S: the setup cost.

Given these notations, we assume that there is a decision maker who procures an order quantity of Q units of a product per cycle. This order quantity of Q units will be inspected, and we assume Qr units of the order quantity will meet the quality requirements (i.e., Q is the prior order quantity while Qr is the posterior order quantity). The remaining $Q(1-r)$ units that do not meet the quality requirements are assumed to be discarded without any cost/value to the decision maker. The Qr units meeting the quality requirements will be sold to customers at P per unit.

In this paper, the relationship between the fraction r and K is characterized by

$K(r)$ function, which is differentiable and increasing with respect to r. That is, we are assuming that, by investing more in equipment and training, the fraction r can be increased. Also, we are assuming that r is the decision variable (of course $r(K)$ function, where K is a decision variable, is also feasible). Furthermore, for our analysis, we assume that, if ROI is non-positive for an ROI maximizing decision maker, the decision maker ceases to operate. Therefore, we focus on the case of positive ROI.

Finally, the following simplifying assumptions are made throughout this paper, which are often utilized in EOQ-type papers (e.g., Morse and Scheiner 1979).
(1) Shortage is not allowed. (2) The sales quantity per unit time and selling price per unit are deterministic and constant over time.

2.2 Optimality conditions for Problem X

In this paper, we consider two types of the ROI maximization problems of Problem X and Problem Y. Under Problem X, ROI is maximized over Q given the current level of the investment in equipment and training, K_F, and the corresponding fraction of an order quantity meeting the quality requirements, r_F (i.e., $K_F = K(r_F)$). That is, the investment in equipment and training and the fraction of an order quantity meeting the quality requirements are assumed to be fixed. The total cost per unit time, TC, consists of the setup cost, the variable cost, and the holding cost and the investment in equipment and training. Given the posterior order quantity, Qr, the cycle length is expressed as $\frac{Qr}{D}$. Hence, mathematically, we have: $TC = \frac{SD}{Qr_F} + \frac{CD}{r_F} + \frac{ICQr_F}{2} + K_F$.

Since the total revenue per unit time is the selling price per unit multiplied by the

sales quantity per unit time (i.e., PD), the profit per unit time, Π, is obtained by subtracting the total cost per unit time from the revenue per unit time. i.e., $\Pi = PD - TC$.

Let us now formally define ROI. Traditionally, ROI is defined to be the ratio of the profit per unit time over the average investment per unit time (see e.g., Schroeder and Krishnan 1976). In our model, the average investment consists of the average inventory investment and the average investment in equipment and training. Mathematically, the average inventory investment is given by $\frac{CQr}{2}$ (i.e., only the fraction of an order quantity meeting the quality requirements will be stored as inventory). On the other hand, the average investment in equipment and training is given by K. Hence, ROI given by K_F is as follows:

$$R_F = (PD - \frac{SD}{Qr_F} - \frac{CD}{r_F} - \frac{ICQr_F}{2} - K_F)/(\frac{CQr_F}{2} + K_F) \tag{1}$$

Since ROI is maximized over the order quantity, an equivalent model formulation (see Luenberger 1984) for Problem X is given by

$$\textbf{Problem X:} \quad \min_{Q>0} \; -R_F \tag{2}$$

Then, the first order necessary condition (FONC) for Problem X is

$$\frac{SD}{Q^2r} - \frac{ICr}{2} - \frac{RCr}{2} = 0 \tag{3}$$

From the FONC (3), we obtain the following equation:

$$Q_F^* = [CDSr_F + (2CDK_FSr_F\mathcal{M}_F + C^2D^2S^2r_F^2)^{0.5}]/(Cr_F\mathcal{M}_F) \tag{4}$$

where $\mathcal{M}_F = PDr_F - CD - K_Fr_F + IK_Fr_F$. It can be verified that Q_F^* is unique and satisfies the second order sufficient condition (SOSC) at optimality. i.e.,

$$\frac{2SD}{Q^3r}/(\frac{CQr}{2} + K) > 0 \tag{5}$$

Hence, Q_F^* is the unique global optimal solution for Problem X and the corresponding ROI, R_F^*, is the global optimal ROI.

2.3 Optimality conditions for Problem Y

For Problem Y, the decision maker has an option to invest additional money in quality improvement. Hence, under Problem Y, ROI is maximized over Q as well as r for $0 < r_{min} \leq r \leq r_{max} < 1$ where r_{min} represents the current level of r, r_F, while r_{max} represents the technologically feasible maximum fraction of an order quantity meeting the quality requirements. We will denote the corresponding investment in equipment and training $K(r_{min})$ and $K(r_{max})$ as K_{min} and K_{max}, respectively. An equivalent model formulation (see Luenberger 1984) for Problem Y is given below.

$$\textbf{Problem Y:} \quad \min_{Q>0,\, r} \; -R = \left(\frac{SD}{Qr} + \frac{CD}{r} + \frac{ICQr}{2} + K - PD\right)/\left(\frac{CQr}{2} + K\right) \tag{6}$$

$$\text{subject to } r_{min} - r \leq 0 \text{ and } r - r_{max} \leq 0.$$

From the FONC when $r = r_{min}$ at optimality, we have

$$Q_{min} = [CDSr_{min} + (2CDK_{min}Sr_{min}\mathcal{M}_{min} + C^2D^2S^2r_{min}^2)^{0.5}]/(Cr_{min}\mathcal{M}_{min}) \tag{7}$$

$$-\frac{SD}{Q_{min}r_{min}^2} + \frac{CIQ_{min}}{2} - \frac{CD}{r_{min}^2} + K'_{min} + R_{min}\left(\frac{CQ_{min}}{2} + K'_{min}\right) \geq 0 \tag{8}$$

where $K'_{min} = \frac{\partial K}{\partial r}$ evaluated at $r = r_{min}$ and $\mathcal{M}_{min} = PDr_{min} - CD - K_{min}r_{min} + IK_{min}r_{min}$ while R_{min} is ROI evaluated at $Q = Q_{min}$ and $r = r_{min}$.

In this case, it is easily verified that the SOSC is satisfied. Let us denote this boundary local optimal solution by (Q_{min}^*, r_{min}^*) and the corresponding ROI by R_{min}^* $(= R(Q_{min}^*, r_{min}^*))$.

Likewise, from the FONC when $r = r_{max}$ at optimality, we have

$$Q_{max} = [CDSr_{max} + (2CDK_{max}Sr_{max}\mathcal{M}_{max} + C^2D^2S^2r_{max}^2)^{0.5}]/(Cr_{max}\mathcal{M}_{max}) \quad (9)$$

$$-\frac{SD}{Q_{max}r_{max}^2} + \frac{CIQ_{max}}{2} - \frac{CD}{r_{max}^2} + K'_{max} + R_{max}(\frac{CQ_{max}}{2} + K'_{max}) \leq 0 \quad (10)$$

where $K'_{max} = \frac{\partial K}{\partial r}$ evaluated at $r = r_{max}$ and $\mathcal{M}_{max} = PDr_{max} - CD - K_{max}r_{max} + IK_{max}r_{max}$ while R_{max} is ROI evaluated at $Q = Q_{max}$ and $r = r_{max}$. In this case, it is also easily verified that the SOSC is satisfied. Let us denote this boundary local optimal solution by (Q^*_{max}, r^*_{max}) and the corresponding ROI by R^*_{max} $(= R(Q^*_{max}, r^*_{max}))$.

Finally, let us consider the case when $r = r_{int} \in (r_{min}, r_{max})$ at optimality. We denote the corresponding investment in equipment and training $K(r_{int})$ by K_{int}. Then, we have

$$Q_{int} = [CDSr_{int} + (2CDK_{int}Sr_{int}\mathcal{M}_{int} + C^2D^2S^2r_{int}^2)^{0.5}]/(Cr_{int}\mathcal{M}_{int}) \quad (11)$$

$$-\frac{SD}{Q_{int}r_{int}^2} + \frac{CIQ_{int}}{2} - \frac{CD}{r_{int}^2} + K'_{int} + R_{int}(\frac{CQ_{int}}{2} + K'_{int}) = 0 \quad (12)$$

where $K'_{int} = \frac{\partial K}{\partial r}$ evaluated at $r = r_{int}$ and $\mathcal{M}_{int} = PDr_{int} - CD - K_{int}r_{int} + IK_{int}r_{int}$ while R_{int} is ROI evaluated at $Q = Q_{int}$ and $r = r_{int}$. Let us denote an interior local optimal solution by (Q^*_{int}, r^*_{int}) and the corresponding ROI by R^*_{int} $(= R(Q^*_{int}, r^*_{int}))$. At optimality, we assume that the following second order sufficient condition is met for an interior solution in our analysis:

$$\frac{4CSD^2}{Q^3r^4} + \frac{2SDK''}{Q^3r} + \frac{2SDRK''}{Q^3r} > 0 \quad (13)$$

where $K'' = \frac{\partial^2 K}{\partial r^2}$.

In summary, for Problem X, there always exists a unique global optimal solution because there is only one local optimal solution. On the other hand, for Problem Y, further analysis is needed to determine the global optimal solution because there may

be multiple interior and/or boundary local optimal solutions. This is the topic of the next section.

3. Optimality Analysis

3.1 Derivation of global optimal solutions

Under Problem X, it can be easily verified that there exists a unique global optimal solution. Under Problem Y, however, the argument for the unique global optimal solution is no longer straightforward. In this subsection, we will first address (possible) multiple global optimal solutions (In Subsection 3.3, we will address the determination of the unique global optimal solution). Let us first characterize interior local solutions when they exist.

From equations (11) and (12), the following relation can be obtained:

$$Q_{int} = \sqrt{\frac{2DK'_{int}S}{C^2D - CK'_{int}r^2_{int} + CIK'_{int}r^2_{int}}} \tag{14}$$

where $C^2D - CK'_{int}r^2_{int} + CIK'_{int}r^2_{int} > 0$ for an interior optimal solution. Substituting equation (14) into the objective function (6), we see that the optimal ROI is expressed as a function of r only, $R(r)$. Since we have a function of a single variable, all interior optimal solutions can be obtained by simple numerical methods such as Newton's method (see Luenberger 1984). Let us now suppose that there are n ($n \geq 1$) interior local optimal solutions designated by $(Q^{i*}_{int}, r^{i*}_{int})$, $i = 1, \ldots, n$. We denote the corresponding ROIs by R^{i*}_{int}, $i = 1, \ldots, n$.

By considering the two (possible) local boundary optimal solutions characterized by

conditions (7), (8), (9), and (10), we have a total of $n+2$ possible local optimal (interior and boundary) solutions. Hence, these local optimal solutions represent all possible candidates for a global optimal solution, which may not be unique. The existence of a global optimal solution can be shown via real analysis (see e.g., Apostol 1974 on page 83). Let us denote a global optimal solution by (Q_G^*, r_G^*) and the corresponding ROI by R_G^*. Let us also denote a unique global optimal solution by (Q_{UG}^*, r_{UG}^*) and the corresponding ROI by R_{UG}^*. We will utilize the global optimal prior order quantity Q_G^* in the following analysis of inventory reduction.

3.2 Reduction in the prior order quantity

In this subsection, we will examine if the option to invest in quality improvement leads to reduction in the prior order quantity. In order to show this, we will compare the global optimal prior order quantity for Problem X, Q_F^*, with that for Problem Y, Q_G^*.

From the FONC of Problem X, we have

$$Q_F^* = \{(2SD)/r_F^2 C(I + R_F^*)\}^{0.5} \tag{15}$$

where R_F^* is the global optimal ROI at Q_F^* for Problem X. Similarly, from the FONC of Problem Y, we have

$$Q_G^* = \{(2SD)/r_G^{*2} C(I + R_G^*)\}^{0.5} \tag{16}$$

where R_G^* is the global optimal ROI at Q_G^* for Problem Y. Based on (15) and (16), the relationship between the global optimal ROI and the reduction in the prior order quantity is summarized in Proposition 1.

Proposition 1. Reduction in Prior Order Quantity

1) If $R_G^* = R_{min}^*$, then the reduction in the prior order quantity is zero.

2) If $R_G^* = R_{int}^{i*}$, then the reduction in the prior order quantity is $Q_F^* - Q_{int}^{i*}$.

3) If $R_G^* = R_{max}^*$, then the reduction in the prior order quantity is given by $Q_F^* - Q_{max}^*$.

PROOF: Let us suppose that $R_G^* = R_{min}^*$. Then, $Q_G^* = Q_{min}^* = Q_F^*$ so that the level of the prior order quantity remains the same. Let us now suppose that $R_G^* = R_{int}^{i*}$ for any given i. Then, we observe that $C(I + R_F^*) \leq C(I + R_G^*)$ since $R_{int}^{i*} \geq R_F^*$, and $r_F < r_{int}^{i*}$. Therefore, $Q_{int}^{i*} < Q_F^*$. i.e., the prior order quantity is reduced. Likewise, let us suppose that $R_G^* = R_{max}^*$. Then we observe that $C(I + R_F^*) \leq C(I + R_G^*)$ since $R_{max}^* \geq R_F^*$, and $r_F < r_{max}^*$. Therefore, $Q_{max}^* < Q_F^*$. i.e., the prior order quantity is reduced. ■

Hence, with the option to invest additional money in quality improvement, the prior order quantity will be reduced or remain the same. In particular, if the decision maker finds it optimal to invest additional money in quality improvement, the prior order quantity will be always reduced. In the next subsection, we examine the uniqueness of global optimal solutions.

3.3 Uniqueness of global optimal solution

Thus far, for Problem Y, it is possible to have multiple global optimal solutions. In this subsection, we will employ an additional criterion to induce a unique global optimal solution. The additional criterion is: if the levels of ROI are the same, then the global optimal solution with the smallest prior order quantity will be preferred. The rationale is that, given that the same levels of financial performance (i.e., ROI levels), the smallest

prior order quantity is the most preferable due to factors that are external to this model such as inspection resource requirements (e.g., less inspection equipment, facility, and/or space are required for smaller prior order quantity).

In our model formulation for ROI maximization, given the multiple global optimal solutions with R_G^*, we can show that the lowest prior order quantity is associated with the largest investment in equipment and training as follows. Let us suppose that $R_G^* = R_G^{i*} = R_G^{j*}$ where R_G^{i*} and R_G^{j*} are ROIs corresponding to r_G^{i*} and r_G^{j*}, respectively, satisfying $0 < r_{min} \leq r_G^{i*} < r_G^{j*} \leq r_{max} < 1$. Then, since $r_G^{i*} < r_G^{j*}$, $K_G^{i*} < K_G^{j*}$ and $R_G^* = R_G^{i*} = R_G^{j*}$, $Q_G^{i*} > Q_G^{j*}$ from the previous subsection. We now summarize this hierarchical determination of the unique global optimal solution as follows:

If there are more than one global optimal solutions under the ROI maximization as the primary criterion, then the global optimal solution with the largest investment in equipment and training will be the unique global optimal solution under the prior order quantity minimization as the secondary criterion.

3.4 Reduction in inventory

In this subsection, we will further analyze if the option to invest in quality improvement leads to reduction in the posterior order quantity. Reduction in the posterior order quantity leads to reduction in inventory since the level of inventory is based on the level of the posterior order quantity. Similar to Subsection 3.2, we will compare the unique global optimal posterior order quantity for Problem X, $Q_F^* r_F$, with that for Problem Y, $Q_{UG}^* r_{UG}^*$.

From the FONC for Problem X, we have

$$Q_F^* r_F = \{(2SD)/[C(I + R_F^*)]\}^{0.5} \tag{17}$$

Similarly, from the FONC for Problem Y, we have

$$Q_{UG}^* r_{UG}^* = \{(2SD)/[C(I + R_{UG}^*)]\}^{0.5} \tag{18}$$

Let us first assume that there is single unique global optimal solution based on the primary criterion only. Then, from (17) and (18), if $(Q_{UG}^*, r_{UG}^*) \neq (Q_{min}^*, r_{min}^*)$, then $R_{UG}^* > R_F^*$. Hence, $Q_{UG}^* r_{UG}^* < Q_F^* r_F$ and the inventory is reduced. On the other hand, if $(Q_{UG}^*, r_{UG}^*) = (Q_{min}^*, r_{min}^*)$, then $R_{UG}^* = R_F^*$. Hence, $Q_{UG}^* r_{UG}^* = Q_F^* r_F$ and there is no reduction in inventory.

Let us now assume that there are multiple global optimal solutions based on the primary criterion, and one unique global optimal solution is determined based on the secondary criterion of the prior order quantity minimization. Then, if (Q_{min}^*, r_{min}^*) is not a global optimal solution, then $R_{UG}^* > R_F^*$ and the inventory is reduced. On the other hand, if (Q_{min}^*, r_{min}^*) is a global optimal solution, then $R_{UG}^* = R_F^*$ and there is no reduction in inventory because $Q_{UG}^* r_{UG}^* = Q_{min}^* r_{min}^* = Q_F^* r_F$.

Based on these observations, we present the following proposition.

Proposition 2. Reduction in Inventory

Case 1: When single unique global optimal solution is determined by the primary criterion only:

A) If $(Q_{UG}^*, r_{UG}^*) \neq (Q_{min}^*, r_{min}^*)$, then the inventory is reduced by $Q_F^* r_F - Q_{UG}^* r_{UG}^*$.

B) If $(Q_{UG}^*, r_{UG}^*) = (Q_{min}^*, r_{min}^*)$, then there is no reduction in inventory.

Case 2: When there are multiple global optimal solutions by the primary criterion and

one unique global optimal solution is determined by the secondary criterion:

A) If (Q^*_{min}, r^*_{min}) is not a global optimal solution by the primary criterion only, then the inventory is reduced by $Q^*_F r_F - Q^*_{UG} r^*_{UG}$.

B) If (Q^*_{min}, r^*_{min}) is a global optimal solution by the primary criterion only, then there is no reduction in inventory.

PROOF: Let us suppose that single unique global optimal solution is determined by the primary criterion only. Then, if $(Q^*_{UG}, r^*_{UG}) \neq (Q^*_{min}, r^*_{min})$, the inventory is reduced by $Q^*_F r_F - Q^*_{UG} r^*_{UG}$ because $C(I + R^*_F) < C(I + R^*_{UG})$. If $(Q^*_{UG}, r^*_{UG}) = (Q^*_{min}, r^*_{min})$, then $C(I + R^*_F) = C(I + R^*_{UG})$ so that there is no reduction in inventory.

Now, let us suppose that there are multiple global optimal solutions by the primary criterion and one unique global optimal solution is determined by the secondary criterion. If (Q^*_{min}, r^*_{min}) is not a global optimal solution by the primary criterion only, then $C(I + R^*_F) < C(I + R^*_{UG})$ so that the inventory is reduced by $Q^*_F r_F - Q^*_{UG} r^*_{UG}$. Likewise, if (Q^*_{min}, r^*_{min}) is a global optimal solution by the primary criterion only, $C(I + R^*_F) = C(I + R^*_{UG})$ so that there is no reduction in inventory. ■

3.5 Further analysis of unique global optimal solution

In this subsection, we provide an alternative way to determine the unique global optimal solution by utilizing characteristics of local optimality. This method provides managerial insights and does not depend on the actual calculations of ROIs. From the FONC for Problem Y, we have

$$R = (2SD)/(CQ^2r^2) - I \tag{19}$$

Let us first assume that there are local optimal solutions at $K = K^*_{min}$ and $K = K^*_{max}$. i.e., there are two local optimal solutions and both of them are boundary local optimal solutions. We modify equation (19) in order to obtain the following equivalent conditions:

$$R^*_{max} \geq R^*_{min} \iff (2SD)/(CQ^{*2}_{max}r^{*2}_{max}) - I \geq (2SD)/(CQ^{*2}_{min}r^{*2}_{min}) - I$$

$$\iff \Delta r^*_{(min,max)} \leq \Delta Q^*_{(min,max)} \tag{20}$$

where $\Delta r^*_{(min,max)} = \frac{r^*_{max} - r^*_{min}}{r^*_{max}}$ and $\Delta Q^*_{(min,max)} = \frac{Q^*_{min} - Q^*_{max}}{Q^*_{min}}$. Let us denote inequality condition (20) by $C1$. We note that $\Delta r^*_{(min,max)}$ is the rate of change in the fraction of an order quantity meeting the quality requirements due to the increase in investment in equipment and training from $K = K^*_{min}$ to $K = K^*_{max}$ measured from the local optimal investment level K^*_{max}. Similarly, $\Delta Q^*_{(min,max)}$ is the rate of change in the prior order quantity due to the increase in investment in equipment and training from $K = K^*_{min}$ to $K = K^*_{max}$ measured from the local optimal investment level K^*_{min}. Hence, ROI at $K = K^*_{max}$ is greater than or equal to that at $K = K^*_{min}$ if and only if the rate of change in the fraction of an order quantity meeting the quality requirements is less than or equal to that in the prior order quantity.

Thus far, we have shown an alternative way to describe the relation between the two boundary local optimal solutions. Let us now assume that, in addition to the two boundary local optimal solutions, there is only one interior local optimal solution (Q^*_{int}, r^*_{int}) with R^*_{int}. Then, we can derive the following equivalent relations:

$$R^*_{max} \geq R^*_{int} \iff \Delta r^*_{(int,max)} \leq \Delta Q^*_{(int,max)} \tag{21}$$

$$R^*_{int} \geq R^*_{min} \iff \Delta r^*_{(min,int)} \leq \Delta Q^*_{(min,int)} \tag{22}$$

where $\Delta r^*_{(int,max)} = \frac{r^*_{max}-r^*_{int}}{r^*_{max}}$, $\Delta Q^*_{(int,max)} = \frac{Q^*_{int}-Q^*_{max}}{Q^*_{int}}$, $\Delta r^*_{(min,int)} = \frac{r^*_{int}-r^*_{min}}{r^*_{int}}$, and $\Delta Q^*_{(min,int)} = \frac{Q^*_{min}-Q^*_{int}}{Q^*_{min}}$. Let us denote the inequality conditions (21) and (22) by $C2$ and $C3$, respectively.

If there are multiple interior local optimal solutions, inequalities similar to (20), (21), and (22) can be employed to obtain the unique global optimal solution among the interior local optimal solutions only. Let us denote such a solution by (Q^*_{INT}, r^*_{INT}) with R^*_{INT}.

Now, the unique global optimal solution can be determined as follows. If there exist all three types of local optimal solutions, i.e., (Q^*_{min}, r^*_{min}), (Q^*_{INT}, r^*_{INT}), and (Q^*_{max}, r^*_{max}), first examine if $C2$ holds. (1) If $C2$ holds, then examine if $C1$ holds. If $C1$ holds, the unique global optimal solution is (Q^*_{max}, r^*_{max}) with $R^*_{UG} = R^*_{max}$. Otherwise, the unique global optimal solution is $R^*_{UG} = R^*_{min}$. (2) If $C2$ does not hold, then examine if $C3$ holds. If $C3$ holds, then the unique global optimal solution is (Q^*_{INT}, r^*_{INT}) with $R^*_{UG} = R^*_{INT}$. Otherwise, the unique global optimal solution is $R^*_{UG} = R^*_{min}$.

If there exist two types or one type of local optimal solutions (e.g., (Q^*_{INT}, r^*_{INT}) and (Q^*_{max}, r^*_{max})), a similar approach can be used to determine the unique global optimal solution. Therefore, we now have an alternative way to determine the unique global optimal solution.

4. Analysis under a Linear Investment Function

So far, we have assumed fairly general classes of investment functions. In this section, we show additional managerial insights by employing a linear investment function.

Let us suppose that the investment function is $K(r) = \beta r$ where β is a positive constant. We note that an increase in β leads to the upward shift of the investment function. Hence, for a given level of the fraction of an order quantity meeting the quality requirements, the increase in β raises the investment in equipment and training. Furthermore, when β is large, more investment has insignificant impact on the fraction of an order quantity meeting the quality requirements. On the other hand, when β is small, more investment has significant impact on the fraction of an order quantity meeting the quality requirements.

From conditions (7) and (8), the local optimal solution at $r = r^*_{min}$ is characterized by

$$Q^*_{min} = [CDS + (2\beta CDS\mathcal{M}^*_{min} + C^2D^2S^2)^{0.5}]/(C\mathcal{M}^*_{min}) \tag{23}$$

$$r^*_{min} = r_{min} \tag{24}$$

where $-\frac{SD}{Q^*_{min}r^{*2}_{min}} + \frac{CIQ^*_{min}}{2} - \frac{CD}{r^{*2}_{min}} + \beta + R^*_{min}(\frac{CQ^*_{min}}{2} + \beta) \geq 0$ and $\mathcal{M}^*_{min} = PDr^*_{min} - CD - \beta r^{*2}_{min} + \beta I r^{*2}_{min}$.

Also, the local optimal solution at $r = r^*_{int}$ is, from conditions (11) and (12),

$$Q^*_{int}(r^*_{int}) = \sqrt{\frac{2\beta DS}{C^2D - \beta Cr^{*2}_{int}(\beta) + \beta CIr^{*2}_{int}(\beta)}} \tag{25}$$

$$r^*_{int} = \frac{2\beta CDP + \sqrt{2\beta C^2DS[-4\beta C(1-I) + DP^2 + 2CS(1-I)]}}{\beta DP^2 + 2\beta CS(1-I)} \tag{26}$$

We note that the interior local optimal solution given (25) and (26) is unique. In addition, if the local optimal solution is an interior solution, then $DP^2+2C(S-2\beta)(1-I) \geq 0$. Furthermore, the second order sufficient condition given by (13) is always satisfied by the solutions of (25) and (26).

Finally, by conditions (9) and (10), the local optimal solution at $r = r^*_{max}$ is given by

$$Q^*_{max} = [CDS + (2\beta CDS\mathcal{M}^*_{max} + C^2D^2S^2)^{0.5}]/(C\mathcal{M}_{max}) \tag{27}$$

$$r^*_{max} = r_{max} \tag{28}$$

where $-\frac{SD}{Q^*_{max}r^{*2}_{max}}+\frac{CIQ^*_{max}}{2}-\frac{CD}{r^{*2}_{max}}+\beta+R^*_{max}(\frac{CQ^*_{max}}{2}+\beta) \leq 0$ and $\mathcal{M}^*_{max} = PDr^*_{max} - CD - \beta r^{*2}_{max} + \beta I r^{*2}_{max}$.

We now will comprehensively analyze the optimal behavior of R, Q, and K with respect to parameter β. First, it is easily verified that R is a decreasing function with respect to β. Next, let us define the critical value of β, β_1. Mathematically,

$$\beta_1 = \min_{\beta>0} \{\beta\} \quad \text{subject to} \quad R^*_{UG}(\beta) = R^*_{min}(\beta) \tag{29}$$

β_1 defines the minimum β value at which the unique global optimal R^*_{UG} is equal to R^*_{min}. Likewise, we can define three additional critical values of β, β_2, β_A, and β_B. Mathematically,

$$\beta_2 = \max_{\beta>0} \{\beta\} \quad \text{subject to} \quad R^*_{UG}(\beta) = R^*_{max}(\beta) \tag{30}$$

$$\beta_A \in \{\beta | R^*_F(\beta) = 0, \beta > 0\} \tag{31}$$

$$\beta_B \in \{\beta | R^*_{UG}(\beta) = 0, \beta > 0\} \tag{32}$$

It can be verified that β_1, β_2, β_A, and β_B are either uniquely determined or non-existing. Let us first examine the case where all four critical values exist. Then, it can be shown that $\beta_2 < \beta_1$ and $\beta_A \leq \beta_B$. It can also be shown that all possible relative positions of

β_1, β_2, β_A, and β_B are characterized in the following six cases:

If $\beta_1 = \beta_A$, then $\beta_2 < \beta_1 = \beta_A = \beta_B$ (Case (a); see Figure 1.a).

If $\beta_1 < \beta_A$, then $\beta_2 < \beta_1 < \beta_A = \beta_B$ (Case (b); see Figure 1.b).

If $\beta_1 > \beta_A$, then the following four cases may happen.

$\beta_2 < \beta_A < \beta_B < \beta_1$ (Case (c); see Figure 1.c), $\beta_A \leq \beta_2 < \beta_B < \beta_1$ (Case (d); see Figure 1.d), $\beta_A < \beta_B = \beta_2 < \beta_1$ (Case (e); see Figure 1.e), and $\beta_A < \beta_B < \beta_2 < \beta_1$ (Case (f); see Figure 1.f).

First, from Figure 1.a, Figure 1.c, and Figure 1.d, if $\beta \leq \beta_2$, then the decision maker will invest additional money in quality improvement where $K = K^*_{max}$. If $\beta_2 < \beta < \beta_B$, then the decision maker will invest additional money in quality improvement where $K = K^*_{int}$. That is, when $\beta_2 < \beta < \beta_B$, $K = K^*_{max}$ will not be optimal. If $\beta_B \leq \beta$, then the decision maker will cease to operate because the optimal ROI level is not positive. Hence, for Cases (a), (c) and (d), it is never optimal not to invest any additional money in quality improvement.

Also, from Figure 1.b, if $\beta \leq \beta_2$, then the decision maker will invest additional money in quality improvement where $K = K^*_{max}$. If $\beta_2 < \beta < \beta_1$, then the decision maker will invest additional money in quality improvement where $K = K^*_{int}$. That is, when $\beta_2 < \beta < \beta_1$, $K = K^*_{max}$ will not be optimal. If $\beta_1 \leq \beta < \beta_B$, on the other hand, then the decision maker will not invest any additional money in quality improvement. Furthermore, if $\beta_B \leq \beta$, then the decision maker will cease to operate.

Finally, from Figure 1.e and Figure 1.f, if $\beta < \beta_B$, then the decision maker will invest additional money in quality improvement where $K = K^*_{max}$. If $\beta_B \leq \beta$, on the other

hand, then the decision maker will cease to operate. Hence, for Cases (e) and (f), it is only optimal to invest at the maximum in quality improvement.

In addition, for Cases (c), (d), (e), and (f), for $\beta_A \leq \beta < \beta_B$, ROI for Problem X is non-positive while ROI for Problem Y is positive. Hence, by investing additional money in quality improvement, the decision maker will operate with positive ROI (and not cease to operate).

Moreover, as mentioned in Subsection 3.2, if the decision maker finds it optimal to invest additional money in quality improvement, the prior order quantity is always reduced. Hence, the unique global optimal prior order quantity for Problem Y, Q^*_{UG}, is always bounded above by the unique global optimal order quantity for Problem X, Q^*_F. In addition, we observe that the fraction of an order quantity meeting the quality requirements for Problem Y, r^*_{UG} is bounded below by that for Problem X, r_F.

Thus far, we have examined the case where all four critical values exist. We note that similar analyses can be done where some critical values do not exist. The subsequent analyses are simpler because of the absence of some critical values of β. We now proceed to illustrate some of the features in the following numerical example.

Example 1

Let us suppose that $C = \$100$, $D = 25$ per month, $I = 0.1$ per month, $P = \$500$, $S = \$1000$, $r_{min} = 0.65$ and $r_{max} = 0.95$. Then, the four critical values of β_1, β_2, β_A, and β_B are 1072, 243, 12226, and 12226, respectively. The corresponding Q^*_F, $Q^*_F r_F$, R^*_F, Q^*_{UG}, r^*_{UG}, $Q^*_{UG} r^*_{UG}$ and R^*_{UG} are summarized in Table 1.

First, we recognize that this example is Case (b) in Figure 1. Hence, as β increases,

ROIs for Problems X and Y decrease. When $\beta_1 \leq \beta \leq \beta_B$, we observe that ROIs are the same while, when $\beta < \beta_1$, ROI for Problem Y is strictly greater than ROI for Problem X. Furthermore, when $\beta < \beta_2$, the decision maker will invest additional money in quality improvement where $R = R^*_{max}$.

Also, it can be shown that, as β increases, the prior and posterior order quantities for Problem X and Y increase. However, it can be verified that, when $\beta_1 \leq \beta \leq \beta_B$, the order quantities are the same while, when $\beta < \beta_1$, the order quantity for Problem X is strictly greater than that for Problem Y. That is, if $\beta < \beta_1$, then inventory is reduced when there is an option to invest additional investment in quality improvement.

In addition, it can be shown that, when $\beta < \beta_1$, the fraction of an order quantity meeting the quality requirements for Problem Y decreases as β increases. On the other hand, it can also be shown that, when $\beta_1 \leq \beta \leq \beta_B$, the investment level in equipment and training for Problem Y remains the same as that for Problem X, i.e., no additional investment to improve quality is the optimal policy. On the other hand, $\beta \leq \beta_1$, the decision maker will invest additional money in quality improvement. Finally, when $\beta_B \leq \beta$, the decision maker ceases to operate. That is, even if the decision maker invests additional money in quality improvement, nonpositive ROI level results.

Table 1 Sensitivity analysis of change in β.

β	Problem X			Problem Y			
	Q^*_F	$Q^*_F r_F$	R^*_F	Q^*_{UG}	r^*_{UG}	$Q^*_{UG} r^*_{UG}$	R^*_{UG}
243	11.029	7.16885	9.630	7.2684	0.95	6.90498	10.38
1072	16.000	10.4	4.521	16.000	0.65	10.4	4.521
12226	108.75	70.6875	0.000	108.75	0.65	70.6875	0.000

5. Concluding Remarks

In this paper, we constructed and analyzed inventory and investment in quality improvement policies under ROI maximization. Specifically, first, we showed how an ROI maximization problem is formulated. Next, the unique global optimal solution is determined by employing the primary criterion of ROI maximization and the secondary criterion of the prior order quantity minimization.

In addition, we showed how the levels for the prior and posterior order quantities are reduced when it is optimal to invest additional money in quality improvement. Furthermore, we provided an alternative way of determining the unique global optimal solution based on the rates of change in the fraction of an order quantity meeting the quality requirements and the prior order quantity.

Finally, under the assumption of a linear investment function, we first obtained the unique global optimal solution in closed-form. Next, we derived various interesting managerial insights with respect to the critical parameter of β where $\frac{1}{\beta}$ represents the rate of change in the fraction of an order quantity meeting the quality requirements with respect to investment K. Specifically, it is easily verified that the optimal ROI is a decreasing function with respect to β. Hence, the decision to invest, not to invest, or to cease to operate critically depends on the value of β.

There are several extensions that will further enhance the importance and relevance of our model. They include incorporation of more sophisticated features such as shortages, delivery lags, and stochastic demand rates, etc. From the perspective of investing in quality improvement, it would be of interest to study the allocation of the investment

in quality improvement. For example, how much should be invested in purchasing or leasing new equipment and how much should be invested in employees training and wages.

References

Apostol, Tom M. (1974). *Mathematical analysis*, second edition. Reading, MA: Addison-Wesley Publishing Company.

Cheng, T.C.E. (1991). EPQ with process capability and quality assurance considerations. *Journal of Operations Research Society*, *42*(8), 713–720.

Hillier, Frederick S. and Lieberman, Gerald J. (1995). *Introduction to operations research*, sixth edition. New York, New York: McGraw-Hill Publishing Company.

Hong, Jae-Dong and Hayya, Jack C. (1995). Joint investment in quality improvement and setup reduction. *Computers and Operations Research*, 567–574.

Kekre, Sunder and Mukhopadhyay, Tridas (1992). Impact of electronic data interchange technology on quality improvement and inventory reduction programs: A field study. *International Journal of Production Economics*, *28*, 265–282.

Ladany, S. and Sternlieb, A. (1974). The interaction of economic ordering quantities and marketing policies. *AIIE Transactions*, *6*(1), 35–40.

Lee, Hau L. and Rosenblatt Meir J. (1985). Optimal inspection and ordering policies for products with imperfect quality. *IIE Transactions*, *17*(3), 284–289.

Luenberger, David G. (1984). *Linear and nonlinear programming*, second edition. Reading, MA: Addison-Wesley Publishing Company.

Morse, Wayne J and Scheiner, James H. (1979). Cost minimization, return on investment, residual income: alternative criteria for inventory models. *Accounting and Business Research*, *9*(3), 320–324.

Oakleaf, Robert B. (1972). Retail trade ROI. *Management Accounting*, 25–26.

Porteus, Evan L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, 137-144.

Reece, James S and Cool, William R. (1978). Measuring investment center performance. *Harvard Business Review*, *56*(3), 28–180.

Rosenberg, David. (1991). Optimal price-inventory decisions: profit vs. ROII. *IIE Transactions*, *23*(1), 17–22.

Schroeder, Roger G. and Krishnan, Ramakrishnan (1976). Return on investment as a criterion for inventory models. *Decision Sciences*, *7*(4), 697–704.

Smith, W. M. (1958). An investigation of some quantitative relationships between breakeven point analysis and economic lot size theory. *AIIE Transactions*, *9*, 52–57.

Voss, C.A. (1987). *International trends in manufacturing technology: Just-In-Time manufacture.* Linz, Austria: IFS Publications.

Whitin, T. M. (1955). Inventory control and price theory. *Management Science*, *2*, 61–68.

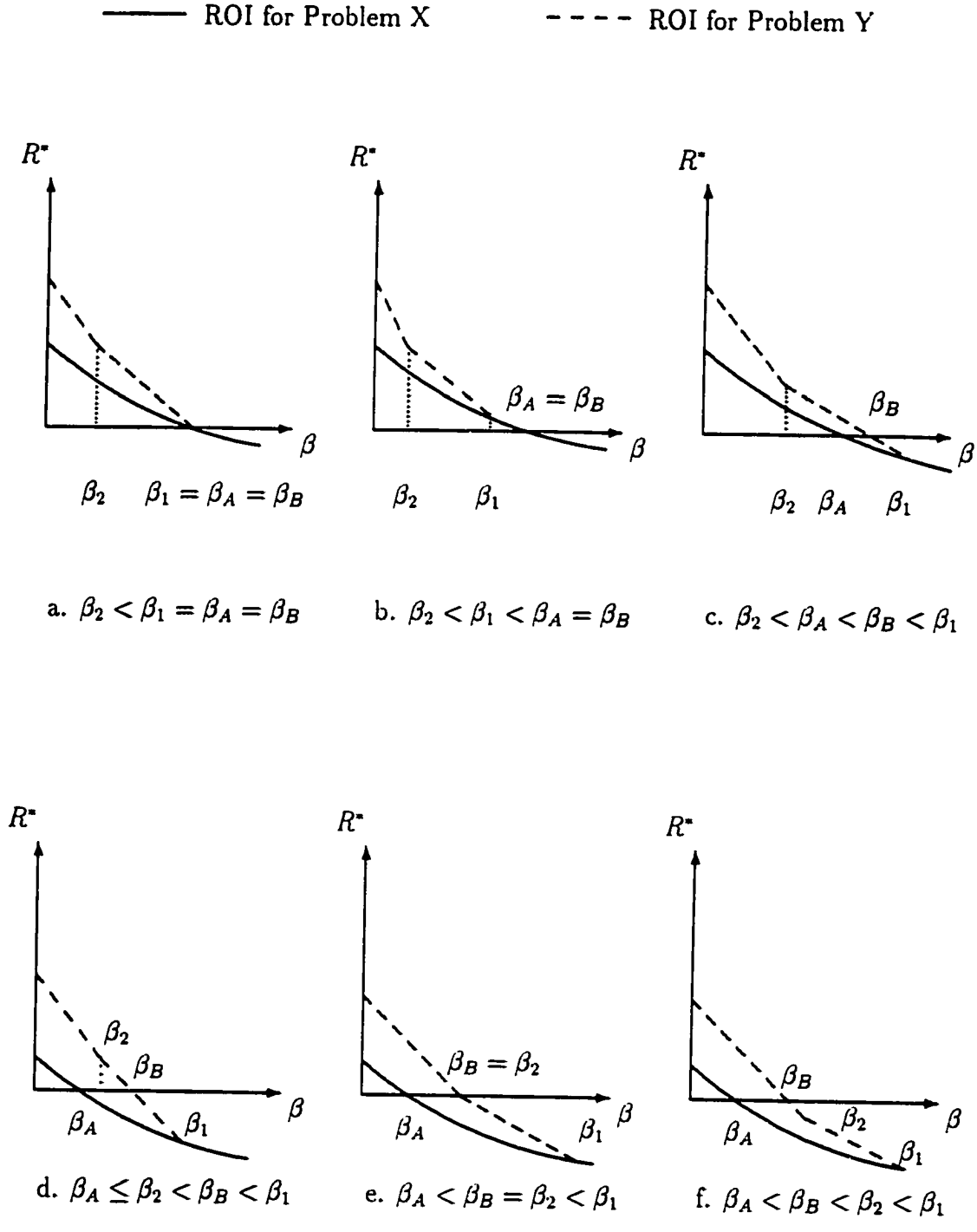

Figure 1 The Optimal ROI vs. β

CHAPTER 2. INVENTORY AND INVESTMENT IN SETUP OPERATIONS UNDER RETURN ON INVESTMENT MAXIMIZATION

A paper prepared for submission to Computers and Operations Research

Toshitsugu Otake and K. Jo Min

Abstract

In this paper, we construct and analyze inventory and investment in setup operations policies under return on investment (ROI) maximization. The key contributing features of this paper are the establishment of an ROI model and characterization of the unique global optimal solution when there exists an option to invest in setup operations. We also show how the inventory level is reduced when it is optimal to invest additional money in setup operations and derive the unique optimal solutions in closed-form when the setup cost is a rational or linear function of the level of investment. Various interesting managerial insights are provided.

1. Introduction

In this paper, we construct and analyze inventory and investment in setup operations policies under return on investment (ROI) maximization for decision makers of inventory systems. ROI is a widely utilized economic performance measure dealing with finished goods inventories (see e.g., Schroeder and Krishnan 1976; Morse and Scheiner 1979; and Reece and Cool 1978). This paper is motivated by the lack of mathematical models with ROI as an economic performance criterion when the option of investing in setup operations exists.

The primary contributions of this paper are: (1) Formulation of the ROI model and characterization of the unique optimal policies consisting of the order quantity and the level of investment in setup operations, (2) Characterization of inventory reduction when it is optimal to invest additional money in setup operations, and (3) Closed-form optimal policies and managerial insights when the setup cost is a rational or linear function of the level of investment.

Traditionally, numerous papers have employed the profit maximization (or cost minimization) as their objective in designing and analyzing inventory models (see e.g., Whitin 1955; Smith 1958; Ladany and Sternlieb 1974; Hillier and Lieberman 1995). Meanwhile, Schroeder and Krishnan (1976) proposes an inventory model under an alternative optimization criterion of ROI maximization. Also, Rosenberg (1991) compares and contrasts profit maximization vs. return on inventory investment with respect to logarithmic concave demand functions.

Thus far, we have reviewed the inventory literature on performance criteria. Let us

now proceed to review the setup investment models as follows. Recently, the superiority of an inventory management system called Zero Inventory (often synonymous with Kanban and Just-in-Time; see e.g., Zangwill 1987) has attracted a great deal of attention not only from industries but also from academia. The essential philosophy of Zero Inventory management system is that inventory results from operational inefficiency. Hence, the higher the level of inventory, the greater the operational inefficiency. From this perspective, it is well known that several Japanese and American producers strive to reduce the level of inventory as much as possible. In order to reduce the level of inventory, meanwhile, numerous experts in industries and academia find it essential to reduce the setup cost of production.

In Porteus (1985), such efforts to reduce the setup cost are mathematically incorporated by introducing an investment cost function of reducing the setup cost to undiscounted EOQ models. For the cases of logarithmic investment cost functions and power investment cost functions, his models demonstrate decreased operational costs when the setup cost is reduced. Porteus (1986a) extends Porteus (1985) to the case of discounted EOQ models. Billington (1987) formulates a model of which setup cost is a function of capital expenses and investigates the relations among holding, setup, and capital expenses. Hong, Xu, and Hayya (1993) proposes a dynamic lot-sizing model of which setup reduction and process quality are functions of capital expenditure. Kim, Hayya, and Hong (1992) investigates several classes of setup reduction functions by employing the economic production quantity model.

We note that, in all these papers in setup investment models, the performance crite-

rion has been the minimization of the cost or the maximization of the profit. Meanwhile, hitherto, there has been no analytical formulation of ROI with an option to invest in setup operations. Since one of the most frequently used criteria in inventory systems other than cost minimization/profit maximization is that of ROI maximization, a comprehensive and quantitative study of ROI maximization is highly desirable. (In deciding the optimal level of investment for setup operations, it would be inherently suboptimal for ROI maximizing decision makers to utilize the existing models constructed for cost-minimizing/profit-maximizing decision makers). The comprehensive and quantitative study is also desirable because the existing literature qualitatively discusses the link between ROI and the inventory reduction (see e.g., Oakleaf 1972).

The rest of this paper is organized as follows. We first formulate the ROI maximization model for inventory and investment in setup operations, and characterize the unique global optimal solution. Next, under the assumption of fairly general classes of setup cost functions, we show how the inventory level is reduced when it is optimal to invest additional money in setup operations. Then, for the specific cases of rational and linear setup cost functions, the optimal closed-form solutions are obtained and several interesting managerial insights are presented. Finally, summary and concluding remarks are made.

2. Model Formulation and Optimality Conditions

2.1 Definitions and assumptions

First, for a decision maker with a single product under ROI maximization, various notations and their definitions used in this paper are as follows:

Q: the order quantity.

C: the variable cost per unit.

I: the inventory holding cost expressed as a fraction of the unit cost per unit time, which excludes the opportunity cost of funds tied up in inventory.

K: the capital investment per unit time in setup operation.

$S(K)$: the setup cost as a function of K.

P: the selling price per unit.

D: the sales quantity per unit time.

Next, the following simplifying assumptions are made throughout this paper (which are often utilized in EOQ-type papers; e.g., Morse and Scheiner 1979).

(1) There are no learning effects in setup or production. (2) Shortage is not allowed. (3) The sales quantity per unit time and selling price per unit are deterministic and constant over time. In addition, as in Billington (1987), we assume that the setup cost $S(K)$ is a decreasing and differentiable function of K. Finally, we assume that, if the profit (hence ROI) is non-positive, then the decision maker stops operating (i.e., the firm ceases to operate). Therefore, we focus on the case of positive profit (hence ROI).

2.2 Optimality conditions for Problem A

In this paper, we consider two types of the ROI maximization problems of Problem A and Problem B. Under Problem A, ROI is maximized over Q given the current level of the capital investment, K_F. i.e., the capital investment level is assumed to be fixed. The total cost per unit time, TC, consists of costs of the setup cost, the variable cost, and the holding cost and the capital investment per unit time in setup operations (see e.g., Billington 1987). Mathematically, the total cost per unit time is expressed as follows: $TC = \frac{S_F D}{Q} + CD + \frac{ICQ}{2} + K_F$ where $S_F = S(K_F)$.

Since the total revenue per unit time is the selling price per unit multiplied by the sales quantity per unit time (i.e., PD), the profit per unit time, Π, is obtained by subtracting the total cost per unit time from the revenue per unit time. i.e., $\Pi = PD - \frac{S_F D}{Q} - CD - \frac{ICQ}{2} - K_F$.

The inventory has been widely viewed as a capital investment for profits (see Schroeder and Krishnan 1976; Morse and Scheiner 1979; Oakleaf 1972) and the capital investment in setup operations is also viewed as an investment. Hence, the average investment per unit time is given by $\frac{CQ}{2} + K_F$.

Since ROI is defined as the ratio of the profit per unit time over the average investment per unit time, ROI given K_F as in Chen (1995) is obtained as follows:

$$R_F = (PD - \frac{S_F D}{Q} - CD - \frac{ICQ}{2} - K_F)/(\frac{CQ}{2} + K_F) \tag{1}$$

Since ROI is maximized over the order quantity, an equivalent model formulation (see Bazaraa et al. 1993; Luenberger 1984) for Problem A is given by

$$\textbf{Problem A:} \quad \min_{Q>0} \; -R_F \tag{2}$$

From the first order necessary condition (FONC), we obtain the following equation:

$$Q_F^* = [CDS_F + (2CDK_F S_F \mathcal{M}_F + C^2 D^2 S_F^2)^{0.5}]/(C\mathcal{M}_F) \quad (3)$$

where $\mathcal{M}_F = PD - CD - K_F + IK_F$. It can be verified that Q_F^* is unique and satisfies the second order sufficient condition (SOSC). Hence, Q_F^* is the unique global optimal solution for Problem A and the corresponding ROI, R_F^*, is the global optimal ROI.

2.3 Optimality conditions for Problem B

For Problem B, the decision maker has an option to invest additional money in setup operations. Hence, under Problem B, ROI is maximized over Q as well as K for $K_{min} \leq K \leq K_{max}$ where K_{min} represents the current level of K, K_F, while K_{max} represents the technologically feasible maximum investment. We will denote the corresponding setup costs $S(K_{min})$ and $S(K_{max})$ as S_{min} and S_{max}, respectively. An equivalent model formulation (see Bazaraa et al. 1993; Luenberger 1984) for Problem B is given below.

Problem B: $$\min_{Q>0,\ K} \quad -R = (\frac{S(K)D}{Q} + CD + \frac{ICQ}{2} + K - PD)/(\frac{CQ}{2} + K) \quad (4)$$

subject to $K_{min} - K \leq 0$ and $K - K_{max} \leq 0$.

From the FONC when $K = K_{min}$ at optimality, we have

$$Q_{min} = [CDS_{min} + (2CDK_{min} S_{min} \mathcal{M}_{min} + C^2 D^2 S_{min}^2)^{0.5}]/(C\mathcal{M}_{min}) \quad (5)$$

$$[(\frac{S'_{min} D}{Q_{min}} + 1)(\frac{CQ_{min}}{2} + K_{min}) + \Pi_{min}]/(\frac{CQ_{min}}{2} + K_{min})^2 \geq 0 \quad (6)$$

where $\mathcal{M}_{min} = PD - CD - K_{min} + IK_{min}$ and Π_{min} is the profit evaluated at $Q = Q_{min}$ and $K = K_{min}$. In this case, it is easily verified that the SOSC is satisfied. Let us denote this boundary local optimal solution by (Q_{min}^*, K_{min}^*) and the corresponding ROI by R_{min}^* $(= R(Q_{min}^*, K_{min}^*))$.

Likewise, from the FONC when $K = K_{max}$ at optimality, we have

$$Q_{max} = [CDS_{max} + (2CDK_{max}S_{max}\mathcal{M}_{max} + C^2D^2S_{max}^2)^{0.5}]/(C\mathcal{M}_{max}) \quad (7)$$

$$[(\frac{S'_{max}D}{Q_{max}} + 1)(\frac{CQ_{max}}{2} + K_{max}) + \Pi_{max}]/(\frac{CQ_{max}}{2} + K_{max})^2 \leq 0 \quad (8)$$

where $\mathcal{M}_{max} = PD - CD - K_{max} + IK_{max}$ and Π_{max} is the profit evaluated at $Q = Q_{max}$ and $K = K_{max}$. In this case, it is also easily verified that the SOSC is satisfied. Let us denote this boundary local optimal solution by (Q^*_{max}, K^*_{max}) and the corresponding ROI by R^*_{max} $(= R(Q^*_{max}, K^*_{max}))$.

Finally, from the FONC when $K_{int} \in (K_{min}, K_{max})$ at optimality and the corresponding setup cost $S(K_{int}) = S_{int}$, we have

$$Q_{int} = [CDS_{int} + (2CDK_{int}S_{int}\mathcal{M}_{int} + C^2D^2S_{int}^2)^{0.5}]/(C\mathcal{M}_{int}) \quad (9)$$

$$[(\frac{S'_{int}D}{Q_{int}} + 1)(\frac{CQ_{int}}{2} + K_{int}) + \Pi_{int}]/(\frac{CQ_{int}}{2} + K_{int})^2 = 0 \quad (10)$$

where $\mathcal{M}_{int} = PD - CD - K_{int} + IK_{int}$ and Π_{int} is the profit evaluated at $Q = Q_{int}$ and $K = K_{int}$. The corresponding SOSC is expressed below.

$$2S_{int}S''_{int} > (S'_{int})^2 \quad (11)$$

We will assume that, for tractable analysis, this SOSC is satisfied for an interior local optimal solution. Let us denote an interior local optimal solution by (Q^*_{int}, K^*_{int}) and the corresponding ROI by R^*_{int} $(= R(Q^*_{int}, K^*_{int}))$.

In summary, for Problem A, there always exists a unique global optimal solution because there is only one local optimal solution. On the other hand, for Problem B, further analysis is needed to determine the global optimal solution because there may be multiple interior and/or boundary local optimal solutions. This is the topic of the next section.

3. Optimality Analysis

3.1 Derivation of global optimal solutions

Under Problem A, it can be easily verified that there exists a unique global optimal solution. Under Problem B, however, the argument for the unique global optimal solution is no longer straightforward. In this subsection, we will first address (possible) multiple global optimal solutions (In Subsection 3.3, we will address the determination of the unique global optimal solution). Let us first characterize interior local solutions when they exist.

From equations (9) and (10) as in Chen (1995), the following relation can be obtained:

$$Q_{int} = (2S_{int} - S'_{int}K_{int})/(P - C) \tag{12}$$

This is considered as a generalized expression of $\frac{2S}{P-C}$ derived by Schroeder and Krishnan (see Schroeder and Krishnan 1976), which does not consider an option to invest additional money in setup operations. Substituting equation (12) into the objective function (4), we see that the optimal ROI is expressed as a function of K only, $R(K)$. Since we have a function of a single variable, all interior optimal solutions can be obtained by numerical methods. Let us now suppose that there are n ($n \geq 1$) interior local optimal solutions designated by $(Q^{i*}_{int}, K^{i*}_{int})$, $i = 1, \ldots, n$. We denote the corresponding ROIs by R^{i*}_{int}, $i = 1, \ldots, n$.

By considering the two (possible) local boundary optimal solutions characterized by conditions (5), (6), (7), and (8), we have a total of $n+2$ possible local optimal (interior

and boundary) solutions. Hence, these local optimal solutions represent all possible candidates for a global optimal solution, which may not be unique. The existence of a global optimal solution can be shown via real analysis (see e.g., Apostol 1974 on page 83). Let us denote a global optimal solution by (Q_G^*, K_G^*) and the corresponding ROI by R_G^*. We will utilize the global optimal order quantity Q_G^* in the following analysis of inventory reduction.

3.2 Analysis of inventory reduction

In this subsection, we will examine if the option to invest in setup operations results in inventory reduction. In order to show this, we will compare the global optimal order quantity for Problem A, Q_F^*, with that for Problem B, Q_G^*.

From the FONC of Problem A, we have

$$Q_F^* = [(2S_F D)/(IC + CR_F^*)]^{0.5} \tag{13}$$

where R_F^* is the global optimal ROI at Q_F^* for Problem A. Similarly, from the FONC of Problem B, we have

$$Q_G^* = [(2S_G^* D)/(IC + CR_G^*)]^{0.5} \tag{14}$$

where S_G^* is the global optimal setup cost at K_G^* and R_G^* is the global optimal ROI for Problem B. Based on (13) and (14), the relationship between the global optimal ROI and the inventory reduction is summarized in Proposition 1.

Proposition 1. (Inventory Reduction)

1) If $R_G^* = R_{max}^*$, then the level of inventory is reduced, and the reduction in the order quantity is given by $Q_F^* - Q_{max}^*$.

2) If $R_G^* = R_{int}^{i*}$, then the level of inventory is reduced, and the reduction in the order quantity is $Q_F^* - Q_{int}^{i*}$.

3) If $R_G^* = R_{min}^*$, then the level of inventory remains the same, and the reduction in the order quantity is zero.

PROOF: Let us suppose that $R_G^* = R_{min}^*$. Then, $Q_G^* = Q_{min}^* = Q_F^*$ so that the level of inventory remains the same. Let us now suppose that $R_G^* = R_{int}^{i*}$ for any given i. Then, we observe that $2S_G^* D < 2S_F D$ since $K_{int}^{i*} > K_F$. Also we observe that $IC + CR_G^* \geq IC + CR_F^*$ since $R_G^* \geq R_F^*$. Therefore, $Q_{int}^{i*} < Q_F^*$. i.e., the level of inventory is reduced. Likewise, let us suppose that $R_G^* = R_{max}^*$. Then, we observe that $2S_G^* D < 2S_F D$ since $K_{max}^* > K_F$. Also we observe that $IC + CR_G^* \geq IC + CR_F^*$ since $R_G^* \geq R_F^*$. Therefore, $Q_{max}^* < Q_F^*$. i.e., the level of inventory is reduced. ■

Hence, with the option to invest additional money in setup operations, the level of inventory will be reduced or remain the same. In particular, if the decision maker finds it optimal to invest additional money in setup operations, the level of inventory will be always reduced. In the next subsection, employing an additional criterion based on Proposition 1, we will characterized the uniqueness of the global optimal solution.

3.3 Uniqueness of global optimal solution

Thus far for Problem B, it is possible to have multiple global optimal solutions. In this subsection, we will employ an additional criterion based on Proposition 1 to induce a unique global optimal solution. The additional criterion is: if the levels of ROI are the same, then the global optimal solution with the lowest level of order quantity will

be preferred. The rationale is that, given that the levels of financial performance are the same, the smallest inventory is the most preferable due to factors that are external to this model (e.g., storage facilities, space, risk of deterioration and obsoleteness, etc.).

In our model formulation for ROI maximization, given the multiple global optimal solutions with R_G^*, we can show that the smallest order quantity is associated with the largest capital investment as follows. Let us suppose that $R_G^* = R_G^{i*} = R_G^{j*}$ where R_G^{i*} and R_G^{j*} are ROIs corresponding to K_G^{i*} and K_G^{j*}, respectively, satisfying $K_{min} \leq K_G^{i*} < K_G^{j*} \leq K_{max}$. Then, since $2S_G^{i*}D > 2S_G^{j*}D$, where S_G^{i*} and S_G^{j*} are the setup costs corresponding K_G^{i*} and K_G^{j*}, respectively. Also, $IC + CR_G^{i*} = IC + CR_G^{j*}$ and $Q_G^{i*} > Q_G^{j*}$ by equation (14). Hence, the unique global optimal ROI, R_{UG}^*, is R_G^{j*} at $K = K_G^{j*}$ rather than R_G^{i*} at $K = K_G^{i*}$. We now summarize this hierarchical determination of the unique global optimal solution as follows:

If there are more than one global optimal solutions under the ROI maximization as the primary criterion, then the global optimal solution with the largest capital investment will be the unique global optimal solution under the order quantity minimization as the secondary criterion.

3.4 Further analysis of unique global optimal solution

In this subsection, we provide an alternative way to determine the unique global optimal solution by utilizing characteristics of local optimality. This method provides managerial insights and does not depend on the actual calculations of ROIs. From the FONC for Problem B, we have

$$R = (2SD)/(CQ^2) - I \tag{15}$$

Let us first assume that there are local optimal solutions at $K = K^*_{min}$ and $K = K^*_{max}$. i.e., there are two local optimal solutions and both of them are boundary local optimal solutions. We modify equation (15) in order to obtain the following equivalent conditions:

$$R^*_{max} \geq R^*_{min} \iff (2S^*_{max}D)/(CQ^{*2}_{max}) - I \geq (2S^*_{min}D)/(CQ^{*2}_{min}) - I$$

$$\iff \Delta\bar{S}^*_{(min,max)} \leq \Delta Q^*_{(min,max)} \tag{16}$$

where $\Delta\bar{S}^*_{(min,max)} = \frac{\bar{S}^*_{min} - \bar{S}^*_{max}}{\bar{S}^*_{min}}$, $\Delta Q^*_{(min,max)} = \frac{Q^*_{min} - Q^*_{max}}{Q^*_{min}}$, $\bar{S}^*_{max} = \frac{S^*_{max}}{Q^*_{max}}$ and $\bar{S}^*_{min} = \frac{S^*_{min}}{Q^*_{min}}$. Let us denote inequality condition (16) by $C1$. We note that $\bar{S}^*_{min}$ and $\bar{S}^*_{max}$ are the per unit setup cost at $K = K^*_{min}$ and $K = K^*_{max}$, respectively. We also note that $\Delta\bar{S}^*_{(min,max)}$ and $\Delta Q^*_{(min,max)}$ are the rates of change in the per unit setup cost and the order quantity due to the increase in the capital investment from $K = K^*_{min}$ to $K = K^*_{max}$, respectively. Hence, ROI at $K = K^*_{max}$ is greater than or equal to that at $K = K^*_{min}$ if and only if the rate of change in the per unit setup cost due to the increase in investment is less than or equal to that in the order quantity.

Thus far, we have shown an alternative way to describe the relation between the two boundary local optimal solutions. Let us now assume that, in addition to the two boundary local optimal solutions, there is only one interior local optimal solution (Q^*_{int}, K^*_{int}) with R^*_{int}. Then, we can derive the following equivalent relations:

$$R^*_{max} \geq R^*_{int} \iff \Delta\bar{S}^*_{(int,max)} \leq \Delta Q^*_{(int,max)} \tag{17}$$

$$R^*_{int} \geq R^*_{min} \iff \Delta\bar{S}^*_{(min,int)} \leq \Delta Q^*_{(min,int)} \tag{18}$$

where $\Delta\bar{S}^*_{(int,max)} = \frac{\bar{S}^*_{int}-\bar{S}^*_{max}}{\bar{S}^*_{int}}$, $\Delta Q^*_{(int,max)} = \frac{Q^*_{int}-Q^*_{max}}{Q^*_{int}}$, $\Delta\bar{S}^*_{(min,int)} = \frac{\bar{S}^*_{min}-\bar{S}^*_{int}}{\bar{S}^*_{min}}$, and $\Delta Q^*_{(min,int)} = \frac{Q^*_{min}-Q^*_{int}}{Q^*_{min}}$. Let us denote the inequality conditions (17) and (18) by $C2$ and $C3$, respectively.

If there are multiple interior local optimal solutions, inequalities similar to (16), (17), and (18) can be employed to obtain the unique global optimal solution among the interior local optimal solutions only. Let us denote such a solution by (Q^*_{INT}, K^*_{INT}) with R^*_{INT}.

Now, the unique global optimal solution can be determined as follows. If there exist all three types of local optimal solutions, i.e., (Q^*_{min}, K^*_{min}), (Q^*_{INT}, K^*_{INT}), and (Q^*_{max}, K^*_{max}), first examine if $C2$ holds. (1) If $C2$ holds, then examine if $C1$ holds. If $C1$ holds, the unique global optimal solution is (Q^*_{max}, K^*_{max}) with $R^*_{UG} = R^*_{max}$. Otherwise, the unique global optimal solution is $R^*_{UG} = R^*_{min}$. (2) If $C2$ does not hold, then examine if $C3$ holds. If $C3$ holds, then the unique global optimal solution is (Q^*_{INT}, K^*_{INT}) with $R^*_{UG} = R^*_{INT}$. Otherwise, the unique global optimal solution is $R^*_{UG} = R^*_{min}$.

If there exist two types or one type of local optimal solutions (e.g., (Q^*_{INT}, K^*_{INT}) and (Q^*_{max}, K^*_{max})), a similar approach can be used to determine the unique global optimal solution. Therefore, we now have an alternative way to determine the unique global optimal solution, which will be utilized in the next section.

4. Analysis under a Special Setup Cost Function

So far, we have assumed fairly general classes of setup cost functions. In this section, we show additional managerial insights by employing two special setup cost functions. Namely, a rational setup cost function and a linear setup cost function.

4.1 Analysis under a rational setup cost function

Let us suppose that, in this subsection, the setup cost function is a rational function, $S(K) = \frac{\gamma}{K}$ where γ is a positive constant and represents the magnitude of the setup cost (see Ladany and Sternlieb 1974; Chen 1995). We note that an increase in γ leads to the upward shift of the setup cost function. Hence, for a given level of capital investment, the increase in γ raises the setup cost.

From conditions (5) and (6), the local optimal solution at $K = K^*_{min}$ is characterized by

$$Q^*_{min} = [\frac{\gamma CD}{K^*_{min}} + (2\gamma CD\mathcal{M}^*_{min} + \frac{\gamma^2 C^2 D^2}{K^{*2}_{min}})^{0.5}]/(C\mathcal{M}^*_{min}) \tag{19}$$

$$K^*_{min} = K_{min} \tag{20}$$

where $\frac{(\frac{-\gamma D}{Q^*_{min}K^{*2}_{min}}+1)(\frac{CQ^*_{min}}{2}+K^*_{min})+\Pi^*_{min}}{(\frac{CQ^*_{min}}{2}+K^*_{min})^2} \geq 0$ and $\mathcal{M}^*_{min} = PD - CD - K^*_{min} + IK^*_{min}$.

Also, the local optimal solution at $K = K^*_{int}$ is, from conditions (9) and (10),

$$Q^*_{int}(\gamma) = (12\gamma D(P-C))/(-9C\gamma(1-I) + \sqrt{\mathcal{E}}) \tag{21}$$

$$K^*_{int}(\gamma) = (-9C\gamma(1-I) + \sqrt{\mathcal{E}})/(4D(P-C)^2) \tag{22}$$

where $\mathcal{E} = 81C^2\gamma^2(1-I)^2 + 24CD^2\gamma(P-C)^3$. We note that the interior local optimal solution given (21) and (22) is unique.

Finally, by conditions (7) and (8), the local optimal solution at $K = K^*_{max}$ is given by

$$Q^*_{max} = [\frac{\gamma CD}{K^*_{max}} + (2\gamma CD\mathcal{M}^*_{max} + \frac{\gamma^2 C^2 D^2}{K^{*2}_{max}})^{0.5}]/(C\mathcal{M}^*_{max}) \tag{23}$$

$$K^*_{max} = K_{max} \tag{24}$$

where $\frac{(\frac{-\gamma D}{Q^*_{max}K^{*2}_{max}}+1)(\frac{CQ^*_{max}}{2}+K^*_{max})+\Pi^*_{max}}{(\frac{CQ^*_{max}}{2}+K^*_{max})^2} \leq 0$ and $\mathcal{M}^*_{max} = PD - CD - K^*_{max} + IK^*_{max}$.

Let us now derive the procedure for the unique global optimal solution. When

there exist all three types of local optimal solutions, (Q^*_{min}, K^*_{min}), (Q^*_{int}, K^*_{int}), and (Q^*_{max}, K^*_{max}), then from Subsection 3.4,

$$\Delta\bar{S}^*_{(int,max)} - \Delta Q^*_{(int,max)} = \frac{(P-C)K^*_{int}}{(P+\delta_1-C)K^*_{max}} \times \{1 - (\frac{P+\delta_1-C}{P-C})^2 \frac{K^*_{max}}{K^*_{int}}\} \quad (25)$$

where $\delta_1 = \frac{\mu_2^*(\frac{cQ^*_{max}}{2}+K^*_{max})K^*_{max}}{D} > 0$. Hence, the right hand side of equation (25) is negative.

Likewise,

$$\Delta\bar{S}^*_{(min,int)} - \Delta Q^*_{(min,int)} = \frac{(P-\delta_2-C)K^*_{min}}{(P-C)K^*_{int}}\{1 - (\frac{P-C}{P-\delta_2-C})^2 \frac{K^*_{int}}{K^*_{min}}\} \quad (26)$$

where $\delta_2 = \frac{\mu_1^*(\frac{cQ^*_{min}}{2}+K^*_{min})K^*_{min}}{D} > 0$ and the right hand side of equation (26) is negative. The negative values of (25) and (26) imply that, given the three types of the local optimal solutions, (Q^*_{max}, K^*_{max}) is the unique global optimal solution.

From similar analyses over two types and one type of local optimal solutions, we conclude that, if there exists the local optimal solution (Q^*_{max}, K^*_{max}), then (Q^*_{max}, K^*_{max}) is the unique global optimal solution with $R^*_{UG} = R^*_{max}$. Otherwise, whenever there exists the local optimal solution (Q^*_{int}, K^*_{int}), then (Q^*_{int}, K^*_{int}) is the unique global optimal solution with $R^*_{UG} = R^*_{int}$. If there does not exist (Q^*_{int}, K^*_{int}), either, then (Q^*_{min}, K^*_{min}) is the unique global optimal solution with $R^*_{UG} = R^*_{min}$.

This implies that, with the rational setup cost function $S(K) = \frac{\gamma}{K}$, there is always only one global optimal solution under the ROI maximization criterion. i.e., there is no need for the order quantity minimization criterion as the secondary criterion. We will show that this is not the case in Subsection 4.2 with the linear setup cost function.

We note that, for any given set of values for parameters, the feasible set for Problem

A is a subset of the feasible set for Problem B. Hence, the unique global optimal ROI for Problem B, R^*_{UG}, is always bounded below by the unique global optimal ROI for Problem A, R^*_F. In addition, it can be verified that $\frac{\partial R^*_F}{\partial \gamma} < 0$ and $\frac{\partial R^*_{UG}}{\partial \gamma} < 0$. We note that there are three different expressions for R^*_{UG} according to the three cases of $K^*_{UG} = K^*_{min}$, K^*_{int}, and K^*_{max}.

We now will comprehensively analyze the optimal behavior of R, Q, and K with respect to parameter γ. Let us first define the critical value of γ, γ_1. Mathematically,

$$\gamma_1 = \max_{\gamma>0} \{\gamma\} \qquad \text{subject to} \qquad R^*_{UG}(\gamma) = R^*_{min}(\gamma) \tag{27}$$

γ_1 defines the maximum γ value at which the unique global optimal R^*_{UG} is equal to R^*_{min}. Likewise, we can define three additional critical values of γ, γ_2, γ_A, and γ_B. Mathematically,

$$\gamma_2 = \min_{\gamma>0} \{\gamma\} \qquad \text{subject to} \qquad R^*_{UG}(\gamma) = R^*_{max}(\gamma) \tag{28}$$

$$\gamma_A \in \{\gamma | R^*_F(\gamma) = 0, \gamma > 0\} \tag{29}$$

$$\gamma_B \in \{\gamma | R^*_{UG}(\gamma) = 0, \gamma > 0\} \tag{30}$$

It can be verified that γ_1, γ_2, γ_A, and γ_B are either uniquely determined or non-existing. Let us first examine the case where all four critical values exist. Then, it can be shown that $\gamma_1 < \gamma_2$ and $\gamma_A \leq \gamma_B$. It can also be shown that all possible relative positions of γ_1, γ_2, γ_A, and γ_B are characterized in the following six cases:

If $\gamma_1 = \gamma_A$, then $\gamma_1 = \gamma_A = \gamma_B < \gamma_2$ (Case (a); see Figure 1.a).

If $\gamma_1 > \gamma_A$, then $\gamma_A = \gamma_B < \gamma_1 < \gamma_2$ (Case (b); see Figure 1.b).

If $\gamma_1 < \gamma_A$, then the following four cases may happen.

$\gamma_1 < \gamma_A < \gamma_B < \gamma_2$ (Case (c); see Figure 1.c), $\gamma_1 < \gamma_A < \gamma_B = \gamma_2$ (Case (d); see Figure 1.d), $\gamma_1 < \gamma_A < \gamma_2 < \gamma_B$ (Case (e); see Figure 1.e), and $\gamma_1 < \gamma_2 < \gamma_A < \gamma_B$ (Case

(f); see Figure 1.f).

First, from Figure 1.a and Figure 1.b, if $\gamma < \gamma_A$, then the decision maker will not invest additional money in setup operations. If $\gamma \geq \gamma_A$, then the decision maker will cease to operate because the optimal ROI level is not positive. Hence, for Cases (a) and (b), it is never optimal to invest any additional money.

Also, from Figure 1.c and Figure 1.d, if $\gamma \leq \gamma_1$, then the decision maker will not invest additional money in setup operations. If $\gamma_1 < \gamma < \gamma_B$, then the decision maker will invest additional money in setup operations where $K = K^*_{int}$. That is, when $\gamma_1 < \gamma < \gamma_B$, $K = K^*_{max}$ will not be optimal. If $\gamma \geq \gamma_B$, on the other hand, then the decision maker will cease to operate. Hence, for Cases (c) and (d), it is never optimal to invest at the maximum level of $K = K_{max}$.

Finally, from Figure 1.e and Figure 1.f, if $\gamma \leq \gamma_1$, then the decision maker will not invest additional money in setup operations. If $\gamma_1 < \gamma < \gamma_2$, then the decision maker will invest additional money in setup operations where $K = K^*_{int}$. If $\gamma_2 \leq \gamma < \gamma_B$, then the decision maker will invest the maximum where $K = K^*_{max}$. If $\gamma_B \leq \gamma$, on the other hand, then the decision maker will cease to operate.

In addition, for Cases (c), (d), (e), and (f), for $\gamma_A \leq \gamma < \gamma_B$, ROI for Problem A is non-positive while ROI for Problem B is positive. Hence, by investing additional money in setup operations, the decision maker will operate with positive ROI (and not cease to operate).

Moreover, as mentioned in Subsection 3.2, if the decision maker finds it optimal to invest additional money in setup operations, the level of inventory is always reduced.

Hence, the unique global optimal order quantity for Problem B, Q^*_{UG}, is always bounded above by the unique global optimal order quantity for Problem A, Q^*_F. Also, it can be shown that $\frac{\partial Q^*_F}{\partial \gamma} > 0$ and $\frac{\partial Q^*_{UG}}{\partial \gamma} > 0$. In addition, we observe that the capital investment for Problem B, K^*_{UG} is bounded below by that for Problem A, K_F. Also, it can be shown that $\frac{\partial K_F}{\partial \gamma} = 0$ and $\frac{\partial K^*_{UG}}{\partial \gamma} > 0$.

Thus far, we have examined the case where all four critical values exist. We note that similar analyses can be done where some critical values do not exist. The subsequent analyses are simpler because of the absence of some critical values of γ. We now proceed to illustrate some of the features in the following numerical example.

Example 1

Let us suppose that C = \$100, D = 25 per month, I = 0.1 per month, P = \$150, K_{min} = \$50 per month, and K_{max} = \$480 per month. We note that these numerical values are identical to these in Chen (1995). However, the numerical example is substantially different here. Our emphasis is on parametric analysis of γ, which was NOT addressed at all in Chen (1995). Then, the four critical values of γ_1, γ_2, γ_A, and γ_B are 934, 5310395, 144010, and 578730, respectively. The corresponding Q^*_F, R^*_F, Q^*_{UG}, K^*_{UG}, and R^*_{UG} are summarized in Table 1 as follows:

Table 1 Sensitivity analysis of change in γ.

	Problem A		Problem B		
γ	Q^*_F	R^*_F	Q^*_{UG}	K^*_{UG}	R^*_{UG}
934	1.12	7.33	1.12	50	7.33
144010	120	0	25.4	340	0.23
578730	459	-0.07	83.5	416	0
5310395	2097	-0.1	597	480	-0.088

First, we recognize that this example is Case (c). Hence, as γ increases, ROIs for Problems A and B decrease. When $\gamma \leq \gamma_1$, we observe that ROIs are the same while, when $\gamma > \gamma_1$, ROI for Problem B is strictly greater than ROI for Problem A.

Also, it can be shown that, as γ increases, the order quantities for Problem A and B increase. It can be verified that, when $\gamma \leq \gamma_1$, the order quantities are the same while, when $\gamma > \gamma_1$, the order quantity for Problem A is strictly greater than that for Problem B. That is, if $\gamma > \gamma_1$, then inventory is reduced when there is an option to invest additional investment in setup operations.

Finally, it can be shown that, when $\gamma > \gamma_1$, the capital investment level for Problem B increases as γ increases. On the other hand, it can also be shown that, when $\gamma \leq \gamma_1$, the capital investment level for Problem B remains the same as that for Problem A. i.e., no additional investment to reduce the setup cost is the optimal policy.

From these observations, we summarize that when γ is relatively small (hence the setup cost is relatively small), then no additional investment is the optimal policy. However, when γ is relatively large (hence the setup cost is relatively large), then additional investment is the optimal policy, resulting in higher ROI and smaller inventory.

4.2 Analysis under a linear setup cost function

In this subsection, let us consider a linear setup cost function, $S(K) = \alpha - \beta K$, where both α and β are positive constants. We note that α is the intercept and β is the slope of this linear function. We further note that $K \in [K_{min}, K_{max}]$ and $K_{max} < \frac{\alpha}{\beta}$. This function is widely observed in the literature (see e.g., Billington 1987; Kim, Hayya,

and Hong 1992; Chen 1995). It can be verified, by checking the second order necessary and sufficient conditions, that the local optimality will be achieved at $K = K_{min}$ and/or at $K = K_{max}$. i.e., there is no interior local optimal solution. This result is consistent with the findings of Billington (1987).

The actual expressions for the boundary local optimal solutions can be straightforwardly obtained from conditions (5), (6), (7), and (8). Let us now derive the procedure for the unique global optimal solution. As mentioned before, since the local optimality is achieved at $K = K_{min}$ and/or at $K = K_{max}$, the unique global optimal solution will be either at $K = K^*_{min}$ or at $K = K^*_{max}$. Hence, when there exist two types of local solutions, (Q^*_{min}, K^*_{min}) and (Q^*_{max}, K^*_{max}), from Subsection 3.4,

$$\begin{aligned} & \Delta \bar{S}^*_{(min,max)} - \Delta Q^*_{(min,max)} \\ = \; & \frac{(P-\delta_2-C)(2\alpha-\beta K^*_{max})}{(P+\delta_1-C)(2\alpha-\beta K^*_{min})} \times \\ & \{1-[\frac{(P+\delta_1-C)(2\alpha-\beta K^*_{min})}{(P-\delta_2-C)(2\alpha-\beta K^*_{max})}]^2 \frac{\alpha-\beta K^*_{max}}{\alpha-\beta K^*_{min}}\} \end{aligned} \tag{31}$$

$\delta_1 = \frac{\mu_2^*(\frac{cQ^*_{max}}{2}+K^*_{max})K^*_{max}}{D} > 0$ and $\delta_2 = \frac{\mu_1^*(\frac{cQ^*_{min}}{2}+K^*_{min})K^*_{min}}{D} > 0$. Hence, the unique global optimal solution depends on the sign of (31). If the sign of (31) is non-positive, then $R^*_{UG} = R^*_{max}$. On the other hand, if the sign of (31) is positive, then $R^*_{UG} = R^*_{min}$. We note that if the sign of (31) is zero, then $R^*_{UG} = R^*_{max}$. The reason is, even though $R^*_{min} = R^*_{max}$, the secondary criterion of the order quantity minimization favors $K = K^*_{max}$ case. If there exists only one boundary local optimal solution, then it is the unique global optimal solution.

We note that, as in Subsection 4.1, R^*_{UG} is always bounded below by R^*_F. In addition,

it can be verified that $\frac{\partial R_F^*}{\partial \alpha} < 0$ and $\frac{\partial R_{UG}^*}{\partial \alpha} < 0$ while $\frac{\partial R_F^*}{\partial \beta} > 0$ and $\frac{\partial R_{UG}^*}{\partial \beta} > 0$. In what follows, we show how a comprehensive analysis of the optimal behavior of R, Q, and K with respect to parameters α and β is done. Our approach here is similar to that in Subsection 4.1. Let us examine the parameters α first, followed by β.

First, we define three critical values of α, α_1, α_A, and α_B as follows:

$$\alpha_1 = \min_{\alpha > \beta K_{max}} \{\alpha\} \qquad \text{subject to } R_{UG}^*(\alpha) = R_F^*(\alpha) \tag{32}$$

$$\alpha_A \in \{\alpha | R_F^*(\alpha) = 0, \alpha > \beta K_{max}\} \tag{33}$$

$$\alpha_B \in \{\alpha | R_{UG}^*(\alpha) = 0, \alpha > \beta K_{max}\} \tag{34}$$

Likewise, we define three critical values of β, β_1, β_A, and β_B as follows:

$$\beta_1 = \max_{0 < \beta < \frac{\alpha}{K_{max}}} \{\beta\} \qquad \text{subject to } R_{UG}^*(\beta) = R_F^*(\beta) \tag{35}$$

$$\beta_A \in \{\beta | R_F^*(\beta) = 0, 0 < \beta < \frac{\alpha}{K_{max}}\} \tag{36}$$

$$\beta_B \in \{\beta | R_{UG}^*(\beta) = 0, 0 < \beta < \frac{\alpha}{K_{max}}\} \tag{37}$$

Let us first assume that all these critical values of α and β exist. Then, it can be shown that all possible relative positions of α_1, α_A, and α_B are as follows:

If $\alpha_1 = \alpha_A$, then $\alpha_1 = \alpha_A = \alpha_B$ (Case (a); see Figure 2.a).

If $\alpha_1 > \alpha_A$, then $\alpha_A < \alpha_B < \alpha_1$ (Case (b); see Figure 2.b).

If $\alpha_1 < \alpha_A$, then $\alpha_1 < \alpha_A = \alpha_B$ (Case (c); see Figure 2.c).

Next, it can be shown that all possible relative positions of β_1, β_A, and β_B are as follows:

If $\beta_1 = \beta_A$, then $\beta_1 = \beta_A = \beta_B$ (Case (d); see Figure 2.d).

If $\beta_1 > \beta_A$, then $\beta_A = \beta_B < \beta_1$ (Case (e); see Figure 2.e).

If $\beta_1 < \beta_A$, then $\beta_1 < \beta_B < \beta_A$ (Case (f); see Figure 2.f).

As we observe from Figure 2.a to Figure 2.f, analysis similar to the one in Subsection 4.1 can be done. e.g., for Cases (b) and (f), for $\alpha_A \leq \alpha < \alpha_B$ and $\beta_B < \beta \leq \beta_A$, respectively, ROI for Problem A is non-positive while ROI for Problem B is positive. Hence, by investing additional money in setup operations, the decision maker will operate with positive ROI (and not cease to operate).

The key observations for the linear function case that are different from those for the rational function case are as follows:

(1) For Cases (c) and (e), at α_1 and β_1, respectively, even though $R^*_{UG} = R^*_F$, the decision maker will choose R^*_{UG} because of the secondary criterion of the order quantity minimization. That is, $Q^*_{UG} < Q^*_F$ and $K^*_{UG} = K_{max} > K^*_F = K_{min}$ even if $R^*_{UG} = R^*_F$ at α_1 and β_1 (Recall: in the rational function case, the secondary criterion is never needed).

(2) For Cases (a), (b), (d), and (f), we observe that the decision maker will either invest the maximum level of capital investment or cease to operate. In the rational function case, from Figure 1.a to Figure 1.f, we observe that such case can never happen. We note that, for the linear function case, investing additional money that is less than the maximum is never optimal. Therefore, the magnitude of change in K due to changes in parameter values (α and β) may be quite drastic relative to the rational function case (the parameter value here is γ).

Thus far, we have examined the case where all these critical values exist. We note that similar analyses can be done where some critical values do not exist. The subsequent analyses are simpler because of the absence of some critical values of α and/or β.

5. Concluding Remarks

In this paper, we constructed and analyzed inventory policies and investment in setup operations policies under ROI maximization. Specifically, we showed how an ROI maximization problem is formulated and the unique global optimal solution is determined. Furthermore, we showed how the inventory level is reduced when it is optimal to invest additional money in setup operations. Also, we provided an alternative way of determining the unique global optimal solution based on the rates of change in the per unit setup cost and the order quantity. Finally, under the specific assumptions of the rational and linear setup cost functions, we obtained the unique global optimal solutions in closed-form and derived various interesting managerial insights with respect to the critical parameters of α, β, and γ.

There are several extensions that will further enhance the importance and relevance of our model. They include incorporation of more sophisticated features such as shortages, delivery lags, and stochastic demand rates, etc. From the perspective of investing in setup operations, it would be of interest to study the allocation of the investment in setup operations. For example, how much should be invested in purchasing or leasing new equipment and how much should be invested in employees training and wages. Finally, it would be of interest to study the effects of investment in setup operations with respect to process quality and capacity (see e.g., Porteus 1986b; Spence and Porteus 1987).

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References

Apostol, Tom M. (1974). *Mathematical analysis*, second edition, Reading, MA: Addison-Wesley Publishing Company.

Bazaraa, Mokhtar S., Sherali, Hanif D., and Shetty, C. M. (1993). *Nonlinear programming: theory and algorithms*, second edition. New York, New York: John Wiley & Sons.

Billington, Peter. J. (1987). The classical economic production quantity model with setup cost as a function of capital expenditure. *Decision Sciences*, *18*, 25–42.

Chen, Cheng-Kang (1995). *Market-based allocation mechanisms for lot-size decision makers and electric power utilities* (Ph.D. dissertation). Ames, Iowa: Iowa State University.

Hillier, Frederick S. and Lieberman, Gerald J. (1995). *Introduction to operations research*, sixth edition. New York, New York:McGraw-Hill Publishing Company.

Hong, Jae-Dong, Xu, S. H., and Hayya, J. C. (1993). Process quality improvement and setup reduction in dynamic lot-sizing. *International Journal of Production Research*, *33*(11), 2693–2708.

Kim, Seung Lae, Hayya, Jack C., and Hong, Jae-Dong. (1992). Setup reduction in the economic production quantity model. *Decision Sciences*, *23*, 500–508.

Ladany, S. and Sternlieb, A. (1974). The Interaction of economic ordering quantities and marketing policies. *AIIE Transactions*, *6*(1), 35–40.

Luenberger, David G. (1984). *Linear and nonlinear programming*, second edition. Reading, MA: Addison-Wesley Publishing Company.

Morse, Wayne J and Scheiner, James H. (1979). Cost minimization, return on investment, residual income: alternative criteria for inventory models. *Accounting and Business Research*, *9*(3), 320–324.

Oakleaf, Robert B. (1972). Retail trade ROI. *Management Accounting*, 25–26.

Porteus, Evan L. (1985). Investing in reduced setups in the EOQ model. *Management Science*, **31**(8), 998–1010.

Porteus, Evan L. (1986a). Investing in new parameter values in the discounted EOQ model. *Naval Research Logistics Quarterly*, *33*, 39–48.

Porteus, Evan L. (1986b). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, *34*, 137–144.

Reece, James S and Cool, William R. (1978). Measuring investment center performance. *Harvard Business Review*, *56*(3), 28–180.

Rosenberg, David. (1991). Optimal price-inventory decisions: profit vs. ROII. *IIE Transactions*, *23*(1), 17–22.

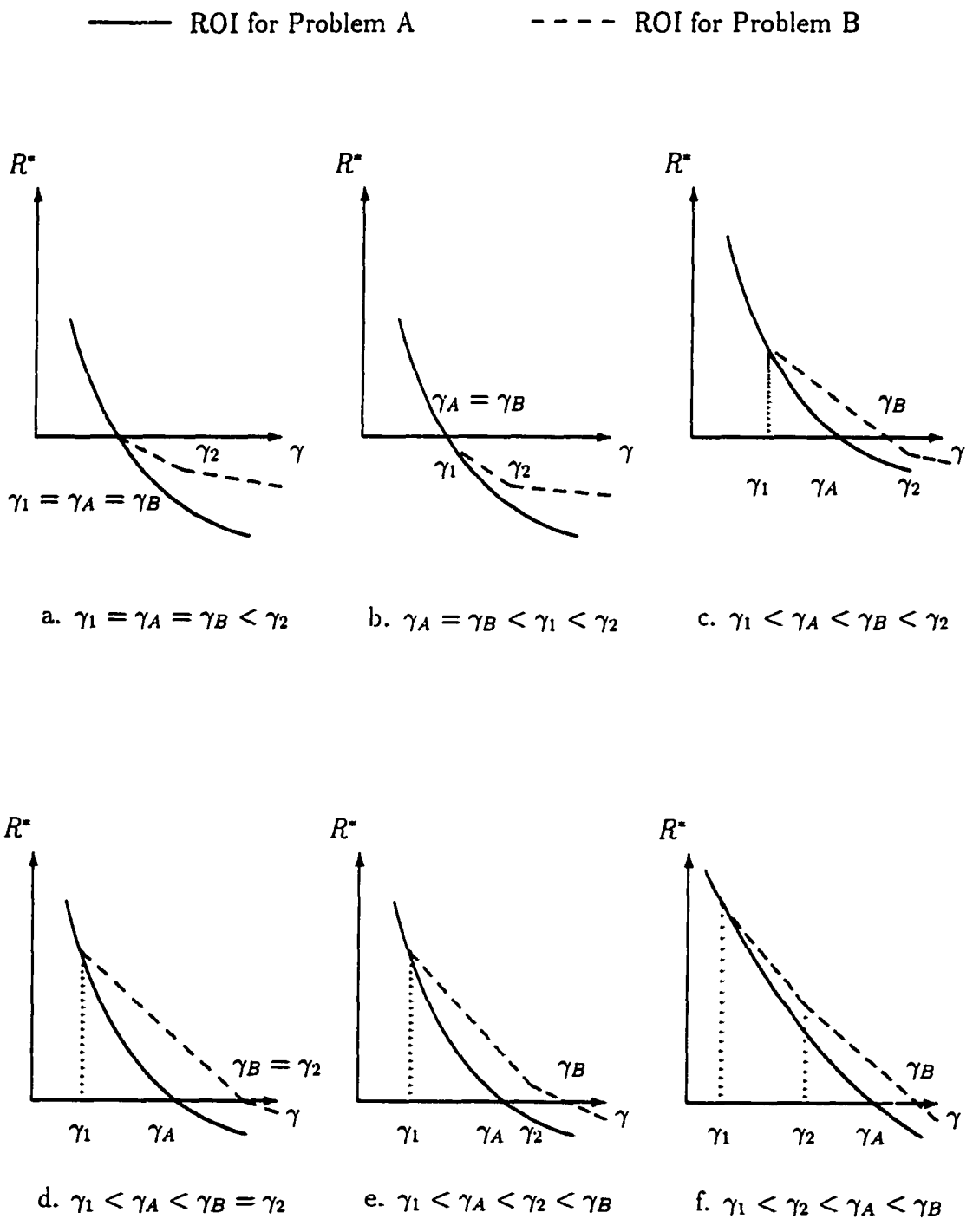
Schroeder, Roger G. and Krishnan, Ramakrishnan. (1976). Return on investment as a criterion for inventory models. *Decision Sciences*, *7*(4), 697–704.

Smith, W. M. (1958). An investigation of some quantitative relationships between breakeven point analysis and economic lot size theory. *AIIE Transactions*, *9*, 52–57.

Spence, A. M., and Porteus, Evan L. (1987). Setup reduction and increased effective capacity. *Management Science*, *33*(10), 1291–1301.

Whitin, T. M. (1955). Inventory control and price theory. *Management Science*, *2*, 61–68.

Zangwill, Willard I. (1987). From EOQ towards ZI. *Management Science*, *33*(10), 1209–1223.



a. $\gamma_1 = \gamma_A = \gamma_B < \gamma_2$ b. $\gamma_A = \gamma_B < \gamma_1 < \gamma_2$ c. $\gamma_1 < \gamma_A < \gamma_B < \gamma_2$

d. $\gamma_1 < \gamma_A < \gamma_B = \gamma_2$ e. $\gamma_1 < \gamma_A < \gamma_2 < \gamma_B$ f. $\gamma_1 < \gamma_2 < \gamma_A < \gamma_B$

Figure 1 The Optimal ROI vs. γ

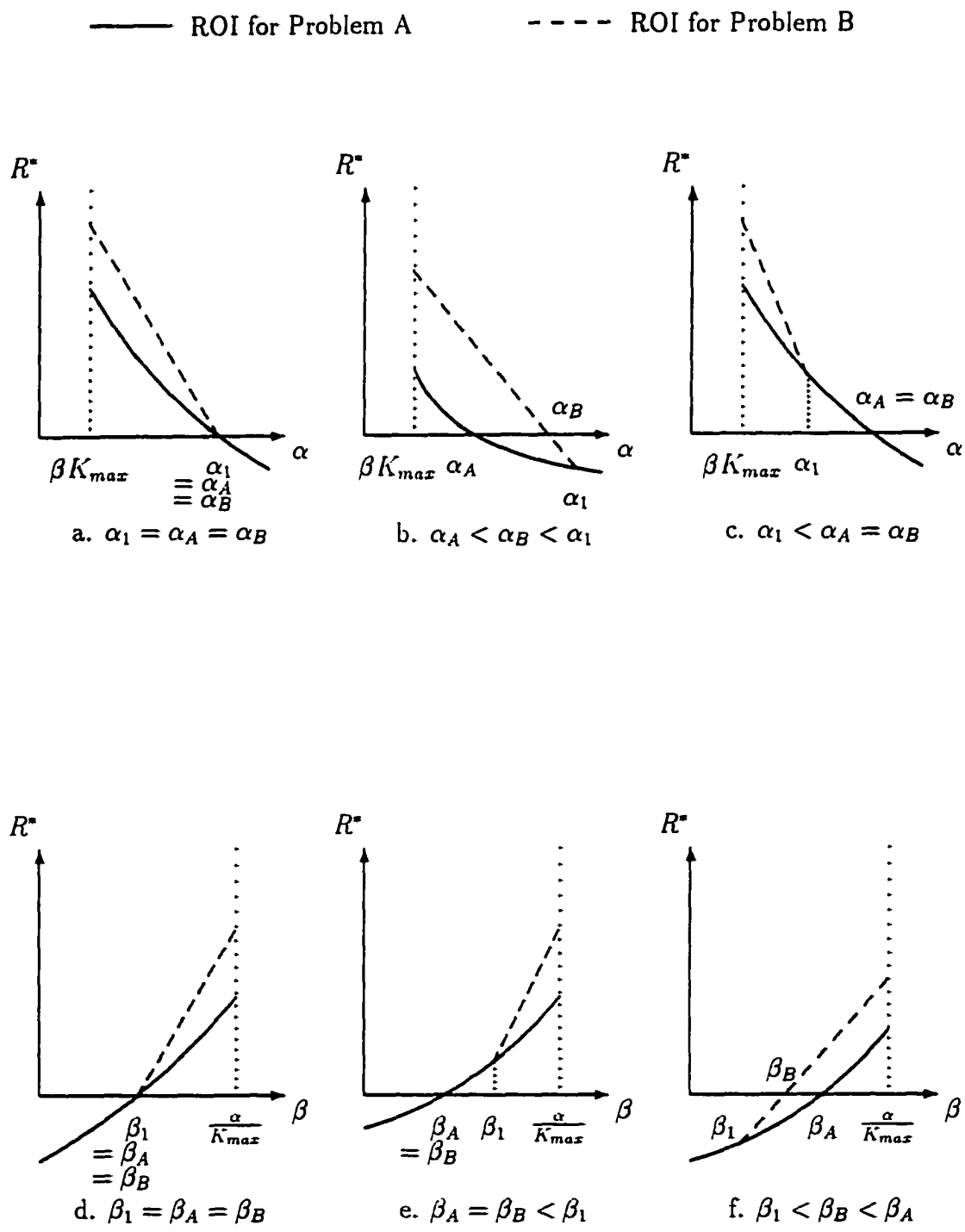


Figure 2 The Optimal ROI vs. α and β

CHAPTER 3. INVENTORY AND CAPITAL INVESTMENT ALLOCATION POLICIES UNDER RETURN ON INVESTMENT MAXIMIZATION

A paper prepared for submission to Engineering Economist

Toshitsugu Otake and K. Jo Min

Abstract

In this paper, we construct and analyze inventory and capital investment allocation policies under return on investment (ROI) maximization. Our model is constructed for a decision maker of a single product with a budget constraint in capital investment. Investment itself can be allocated for reduction of setup cost and/or improvement in quality which is measured by the fraction of non-defective items in a production batch. Interesting managerial insight and a numerical example are provided.

1. Introduction

In this paper, we construct and analyze inventory and capital investment allocation policies under return on investment (ROI) maximization. Our model is constructed for a decision maker of a single product with a budget constraint in capital investment.

Investment itself can be allocated for reduction of setup cost and/or improvement in quality which is measured by the fraction of non-defective items in a production batch.

The decision maker is assumed to determine the production batch size (i.e., the order quantity). This order quantity is then inspected, and only the non-defective items will be stored as inventory while the defective items will be discarded.

The key contributing features of this paper are the establishment of an ROI model with the capital budget constraint. Even though ROI is a widely utilized performance measure in finance and economics (see e.g., Schroeder and Krishnan 1976), the current literature on inventory and investment policies mainly focuses on cost (see e.g., Hong and Hayya 1995). Hence, it is highly desirable to examine such policies under ROI maximization. This is especially true for finished products (see e.g., Morse and Scheiner 1979). For such products, we also derive managerial insights such as how the inventory level is reduced when it is optimal to invest additional money in setup cost reduction and/or quality improvement.

Let us now proceed to review the setup investment models as follows. In Porteus (1985), such efforts to reduce setup cost are mathematically incorporated by introducing an investment cost function of reducing setup cost to undiscounted EOQ models. Porteus (1986) extends Porteus (1985) to the case of discounted EOQ models. By employing the economic production quantity model, Kim, Hayya, and Hong (1992) investigates several classes of setup reduction functions. Leschke and Weiss (1997) analyze investment priorities for setup-reduction programs in a multi-product system. Also, Leschke (1997a) describes the setup-reduction process and Leschke (1997b) provides some guidance of

priority of investment for managers.

Next, the quality issues for a product in an inventory system have been extensively studied. For example, Lee and Rosenblatt (1985) assume imperfect quality of product and examine optimal inspection and ordering policies for products. On the other hand, Cheng (1991) investigates an Economic Production Quantity (EPQ) model with process capability and quality assurance considerations. Since numerous modern production systems emphasize reduction in inventory and improvement in quality, the relation between the quality and inventory reduction is critical for both practitioners and academia. For example, Voss (1987) argues that Just-In-Time production systems lead to increased quality and reduced inventory. In addition, Kekre and Mukhopadhyay (1992) show a negative relationship between inventory and quality by using econometric models.

Moreover, recently, joint investment in setup reduction and quality improvement have been analyzed. Hong, Xu, and Hayya (1993) proposes a dynamic lot-sizing model of which setup reduction and process quality are functions of capital expenditure. Furthermore, Hong and Hayya (1995) examined the trade-offs between investment in setup reduction and investment in quality improvement under cost minimization.

Thus far, we have discussed the literature on setup cost and quality improvement. Let us now proceed with ROI and capital investment in the literature. ROI is one of the most widely used economic and financial performance measure dealing with finished goods inventories as mentioned before (see e.g., Schroeder and Krishnan 1976; Morse and Scheiner 1979; Reece and Cool 1978). Traditionally, there are numerous papers employing the profit maximization or cost minimization as their objective in designing

and analyzing inventory models (see e.g., Whitin 1955; Smith 1958; Ladany and Sternlieb 1974; Hillier and Lieberman 1995). On the other hand, Schroeder and Krishnan (1976) assume an ROI maximization inventory model. Also, by employing logarithmic concave demand functions, Rosenberg (1991) compares and contrasts profit maximization vs. return on inventory investment.

This paper is motivated by the lack of inventory models with the capital budget constraint under ROI maximization when there exists an option to invest in setup operations and quality improvement. Since one of the most widely used economic and financial performance criteria in inventory systems other than profit maximization/cost minimization is ROI maximization, a comprehensive and quantitative study of ROI maximization is highly desirable. The comprehensive and quantitative study is also desirable because the existing literature qualitatively discusses the link between ROI and the inventory reduction (see e.g., Oakleaf 1972).

The remainder of this paper is organized as follows. We first formulate the ROI maximization model for inventory and investment in setup and quality operations. Next, under the assumption of fairly general class of investment function, we show how the inventory level is reduced when it is optimal to invest additional money in setup operations and quality improvement. Moreover, for the specific case of a rational setup cost function with a linear quality improvement function, we illustrate a numerical example to show sensitivity analysis of unit variable cost. Finally, summary and concluding remarks are made.

2. Model Formulation

2.1 Definitions and assumptions

First of all, for a decision maker with a single product, various notations and definitions used throughout this paper are as follows:

Q: the order quantity size prior to inspection.

C: the variable cost per unit.

I: the inventory holding cost expressed as a fraction of the unit cost per unit time, which excludes the opportunity cost of funds tied up in inventory.

K_s: the capital investment per unit time in setup operation.

$S(K_s)$: the setup cost as a function of K_s.

K_r: the capital investment per unit time in quality improvement.

$r(K_r)$: quality level; the fraction of an order quantity meeting the quality requirements, which is a function of K_r.

P: the selling price per unit.

D: the sales quantity per unit time.

Given these notations, we assume that a decision maker determine the order quantity, Q. Also, given the quality level, r, Qr units of the order quantity will meet the quality requirements and they will be stored as inventory (i.e., Q is the prior order quantity while Qr is the posterior order quantity). The remaining defective units, $Q(1-r)$, are assumed to be discarded without any cost/value to the decision maker. The Qr units meeting the quality requirements will be sold to customers at P per unit.

In addition, the following assumptions, utilized in EOQ-type papers (see Morse and Scheiner 1979), are considered in this paper:

(1) Shortage is not allowed.

(2) The sales quantity per unit time and selling price per unit are deterministic and constant over time.

(3) the replenishment rate is infinite.

Furthermore, as in Billington (1987), we assume that the setup cost $S(K_s)$ is a decreasing and differentiable function of K_s. On the other hand, the fraction of an order quantity meeting the quality requirements $r(K_r)$ is an increasing function and differentiable function of K_r. Finally, we assume that, if the profit (hence ROI) is non-positive, then the decision maker stops operating (i.e., the firm ceases to operate). Therefore, we focus on the case of positive profit (hence ROI). In this paper, we consider two types of problems under a return on investment maximization model as described in next subsections.

2.2 ROI maximization model

We consider two types of the ROI maximization problem as Problem A and Problem B. Under Problem A, ROI is maximized over Q given the current level of the investment in setup and quality operations, K_{s_f} and K_{r_f}, respectively. The inventory has been widely viewed as a capital investment for profits (see Schroeder and Krishnan 1976; Morse and Scheiner 1979; Oakleaf 1972) and the capital investments in setup operations and quality improvement are also viewed as an investment. Hence, the average invest-

ment per unit time is given by $\frac{CQr_f}{2} + K_{s_f} + K_{r_f}$. Since ROI is expressed as the ratio of the profit per unit time over the average investment per unit time, ROI, given K_{s_f} and K_{r_f}, is obtained as follows:

$$R_f = (PD - \frac{S_f D}{Qr_f} - \frac{CD}{r_f} - \frac{ICQr_f}{2} - K_{s_f} - K_{r_f})/(\frac{CQr_f}{2} + K_{s_f} + K_{r_f}) \qquad (1)$$

where $S_f = S(K_{s_f})$ and $r_f = r(K_{r_f})$.

Since ROI is maximized over the order quantity, an equivalent model formulation (see Luenberger 1984) for Problem A is given by

$$\textbf{Problem A:} \qquad \min_Q \; -R_f \qquad (2)$$

From the first order necessary condition (FONC; i.e., $\frac{d(-R_f)}{dQ}$, we obtain the following equation:

$$Q_f^* = [CDS_f r_f + (2CD(K_{s_f} + K_{r_f})S_f r_f \mathcal{M}_f + C^2 D^2 S_f^2 r_f^2)^{0.5}]/(Cr_f \mathcal{M}_f) \qquad (3)$$

where $\mathcal{M}_f = PDr_f - CD - (K_{s_f} + K_{r_f})r_f + I(K_{s_f} + K_{r_f})r_f$. It can be verified that Q_f^* is unique and satisfies the second order sufficient condition (SOSC). Hence, Q_f^* is the unique global optimal solution for Problem A and the corresponding ROI, R_f^*, is the global optimal ROI.

Now, under Problem B, we assume that the decision maker has an option to invest additional money in setup cost reduction, quality improvement, or both. This implies that the levels of investment in setup and quality operations are not fixed at the initial levels for Problem A. Hence, ROI for Problem B is expressed as follows:

$$R = (PD - \frac{S(K_s)D}{Qr(K_r)} - \frac{CD}{r(K_r)} - \frac{ICQr(K_r)}{2} - K_s - K_r)/(\frac{CQr(K_r)}{2} + K_s + K_r) \qquad (4)$$

Let us denote the current level of investment in setup operation by $K_{s_{min}}$ and the current level of investment in quality operation by $K_{r_{min}}$. Let us also denote the tech-

nological maximum investment in setup operation by $K_{s_{max}}$ and the technological maximum investment in quality operation by $K_{r_{max}}$ (i.e., more investment is not effective because the technological upper limit has already been reached). Hence, the decision variables are $K_s \in [K_{s_{min}}, K_{s_{max}}]$ where $K_{s_{min}} < K_{s_{max}}$ and $K_r \in [K_{r_{min}}, K_{r_{max}}]$ where $K_{r_{min}} < K_{r_{max}}$.

The current expenditure for both setup operation and quality operation is denoted by $K_{min} = K_{s_{min}} + K_{r_{min}}$. On the other hand, we assume that the decision maker has a capital investment budget of K_{max} ($K_{min} < K_{max}$). Therefore, the budget constraint is given by $K_{min} \leq K_s + K_r \leq K_{max}$. Hence, an equivalent model formulation for Problem B is given below.

$$\textbf{Problem B:} \quad \min_{Q,K_s,K_r} \; -R \tag{5}$$

subject to

$$K_{s_{min}} - K_s \leq 0 \tag{6}$$

$$K_s - K_{s_{max}} \leq 0 \tag{7}$$

$$K_{r_{min}} - K_r \leq 0 \tag{8}$$

$$K_r - K_{r_{max}} \leq 0 \tag{9}$$

$$K_s + K_r - K_{max} \leq 0 \tag{10}$$

The corresponding Lagrangian function, $\mathcal{L}$, is given by $\mathcal{L} = -R + \mu_1(K_{s_{min}} - K_s) + \mu_2(K_s - K_{s_{max}}) + \mu_3(K_{r_{min}} - K_r) + \mu_4(K_r - K_{r_{max}}) + \mu_5(K_s + K_r - K_{max})$. From this function, the corresponding FONCs are:

$$\frac{\partial \mathcal{L}}{\partial Q} = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial K_s} = 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial K_r} = 0 \tag{13}$$

$$\mu_1(K_{s_{min}} - K_s) = 0 \tag{14}$$

$$\mu_2(K_s - K_{s_{max}}) = 0 \tag{15}$$

$$\mu_3(K_{r_{min}} - K_r) = 0 \tag{16}$$

$$\mu_4(K_r - K_{r_{max}}) = 0 \tag{17}$$

$$\mu_5(K_s + K_r - K_{max}) = 0 \tag{18}$$

$$\mu_1 \geq 0 \tag{19}$$

$$\mu_2 \geq 0 \tag{20}$$

$$\mu_3 \geq 0 \tag{21}$$

$$\mu_4 \geq 0 \tag{22}$$

$$\mu_5 \geq 0 \tag{23}$$

$$K_{s_{min}} - K_s \leq 0 \tag{24}$$

$$K_s - K_{s_{max}} \leq 0 \tag{25}$$

$$K_{r_{min}} - K_r \leq 0 \tag{26}$$

$$K_r - K_{r_{max}} \leq 0 \tag{27}$$

$$K_s + K_r - K_{max} \leq 0 \tag{28}$$

We note that the theoretical maximum number of cases, based on bounding/nonbounding

constraints, is $2^5 = 32$. However, some cases are not feasible (e.g., $K_s = K_{s_{min}}$, $K_r = K_{r_{min}}$, and $K_s + K_r = K_{max}$), it can be verified that there are 13 cases that can be optimal.

Now, from $\frac{\partial \mathcal{L}}{\partial Q}$, $\frac{\partial \mathcal{L}}{\partial K_s}$, and $\frac{\partial \mathcal{L}}{\partial K_r}$, we have the following second derivatives:

$$\frac{\partial^2 \mathcal{L}}{\partial Q^2} = \frac{\frac{2SD}{Q^3 r}}{\frac{CQR}{2} + K_s + K_r} \tag{29}$$

$$\frac{\partial^2 \mathcal{L}}{\partial K_s^2} = \frac{\frac{S''D}{Qr}}{\frac{CQR}{2} + K_s + K_r} \tag{30}$$

$$\frac{\partial^2 \mathcal{L}}{\partial K_r^2} = \frac{\frac{2SDr'}{Qr^3} - \frac{SDr''}{Qr^2} + \frac{CQr''(I+R)}{2}}{\frac{CQR}{2} + K_s + K_r} \tag{31}$$

$$\frac{\partial^2 \mathcal{L}}{\partial Q \partial K_s} = \frac{\frac{-S'D}{Q^2 r}}{\frac{CQR}{2} + K_s + K_r} \tag{32}$$

$$\frac{\partial^2 \mathcal{L}}{\partial Q \partial K_r} = \frac{\frac{SDr'}{Q^2 r^2} + \frac{Cr'(I+R)}{2}}{\frac{CQR}{2} + K_s + K_r} \tag{33}$$

$$\frac{\partial^2 \mathcal{L}}{\partial K_s \partial K_r} = \frac{\frac{-S'D}{Qr^2}}{\frac{CQR}{2} + K_s + K_r} \tag{34}$$

From (29), (30), (31), (32), (33), and (34), the Hessian matrix of $\mathcal{L}$, $\mathcal{H}$, is given by

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 \mathcal{L}}{\partial Q^2} & \frac{\partial^2 \mathcal{L}}{\partial Q \partial K_s} & \frac{\partial^2 \mathcal{L}}{\partial Q \partial K_r} \\ \frac{\partial^2 \mathcal{L}}{\partial Q \partial K_s} & \frac{\partial^2 \mathcal{L}}{\partial K_s^2} & \frac{\partial^2 \mathcal{L}}{\partial K_s \partial K_r} \\ \frac{\partial^2 \mathcal{L}}{\partial Q \partial K_r} & \frac{\partial^2 \mathcal{L}}{\partial K_s \partial K_r} & \frac{\partial^2 \mathcal{L}}{\partial K_r^2} \end{bmatrix} \tag{35}$$

Throughout the rest of this paper, we will assume that the second order necessary conditions are satisfied unless otherwise specified (i.e., all principal minors of $\mathcal{H}$ are positive). In next section, we analyze how inventory is reduced when there is an option to invest additional money in setup cost reduction and quality improvement based on Kuhn-Tucker conditions and the second order sufficient conditions.

3. Analysis under General Functions

3.1 Reduction in prior order quantity

Under Problem A, there exists a unique global optimal solution. However, under Problem B, when there may exist multiple local optimal solutions, we cannot argue for uniqueness of global optimal solutions. Let us denote a global optimal solution for Problem B by (Q_G^*, K_G^*), and the corresponding ROI by R_G^*. We will utilize the global optimal order quantity Q_G^* in the following inventory reduction analysis.

In this subsection, we will examine if the option to invest in setup operations and/or quality improvement leads to reduction in the prior order quantity. In order to show this, we will compare the global optimal prior order quantity for Problem A, Q_f^*, with that for Problem B, Q_G^*. From the FONC for both Problem A and Problem B, we can easily see the following Proposition 1:

Proposition 1. (Reduction in Prior Order Quantity)

1) If R_G^* is obtained when $K_s^* = K_{s_{min}}$ and $K_r^* = K_{r_{min}}$, then the reduction in the prior order quantity is zero.

2) Otherwise, the reduction in the prior order quantity is $Q_f^* - Q_G^*$.

We note that the proof is similar to that in Proposition 1 of Chapter 1.

Hence, if the decision maker finds it optimal to invest additional money in setup cost reduction and/or quality improvement, the prior order quantity will be always reduced under the ROI maximization model.

3.2 Reduction in inventory

In this subsection, we will further analyze if the option to invest in setup operations and quality improvement leads to reduction in the posterior order quantity. Reduction in the posterior order quantity leads to reduction in inventory since the level of inventory is based on the level of the posterior order quantity. Similar to Subsection 3.1, from the FONC for both Problem A and Problem B, we can easily see the following Proposition 2:

Proposition 2. (Reduction in Inventory)

1) If R_G^* is obtained when $K_s^* = K_{s_{min}}$ and $K_r^* \geq K_{r_{min}}$, then the reduction in inventory is zero.

2) If R_G^* is obtained when $K_s^* > K_{s_{min}}$ and $K_r^* \geq K_{r_{min}}$, then the reduction in the prior order quantity is $Q_I^* r_I - Q_G^* r_G^*$.

We note that the proof is similar to that in Proposition 1 of Chapter 1.

We note that if we invest additional money in setup cost reduction at optimality, the level of inventory will be reduced. However, even if we invest additional money in quality improvement at optimality, the level of inventory may not be reduced.

3.3 Uniqueness of global optimal solutions

In this subsection, we consider uniqueness of global optimal solutions. That is, for Problem B, since it is possible to have multiple global optimal solutions, we will apply for an additional criterion to induce a unique global optimal solution. The additional criterion is: if the levels of ROI are the same, then the global optimal solution with the smallest inventory, which is similar to the smallest posterior order quantity, will be preferred because of factors that are external to this model (e.g., storage facilities, space, risk of deterioration and obsoleteness, etc). Furthermore, if both the levels of ROI and the smallest levels of posterior order quantity are the same, then the smallest prior order quantity is the most preferable due to factors that are external to this model such as inspection resource requirements (e.g., less inspection equipment, facility, and/or space are required for smaller prior order quantity).

It is easily verified that, given selected multiple global optimal solutions with R_G^*, we can show that the lowest inventory is associated with the largest capital investment in setup operation. In addition, the lowest prior order quantity is associated with the largest capital investment in quality operation. Hence, if there are more than one global optimal solutions under the ROI maximization as the primary criterion, then the global optimal solution with the largest capital investment in setup operation will be the unique global optimal solution under inventory minimization as the secondary criterion. Moreover, if there are more than one global optimal solutions under the ROI maximization as the primary criterion with the largest capital investment in setup operation, then the global optimal solution with the largest capital investment in quality operation will be

the unique global optimal solution under the prior order quantity minimization as the tertiary criterion. In the next section, we provide an illustrative example.

4. Numerical Analysis

Thus far, we constructed ROI maximization problem, analyzed how inventory is reduced when there is an option to invest additional money in setup cost reduction and quality improvement, and examined how to determine the unique global optimal solution if there are multiple optimal solutions. In this section, we employ a rational setup cost function, $S(K_s) = \frac{\gamma}{K_s}$, where γ are positive constant, and a linear quality improvement function, $r(K_r) = \delta K_r$, where δ is positive constant. These functions are widely used in the literature (see e.g., Billington 1987; Kim, Hayya, and Hong 1992).

Now, let us suppose that C = \$100, D = 25 per month, I = 0.1 per month, P = \$500, K_{min} = \$200 per month, K_{max} = \$500 per month, $K_{s_{min}}$ = \$50 per month, $K_{s_{max}}$ = \$400 per month, $K_{r_{min}}$ = \$150 per month, $K_{r_{max}}$ = \$500 per month, $\gamma = 15000$, and $\delta = 0.002$. Since $\gamma = 15000$, $S(K_s) = \frac{15000}{K_s}$ and $\frac{dS}{dK_s} < 0$ over $K_s \in [50, 400]$. Similarly, since $\delta = 0.002$, $r(K_r) = 0.002K_r$ and $\frac{dr}{dK_r} > 0$ over $K_r \in [150, 500]$.

This problem is solved by SAS/IML package (see SAS institute Inc. 1995). First, when initial levels of investment in setup operations and quality improvement are $K_{s_f} = K_{s_{min}} = 50$ and $K_{r_f} = K_{r_{min}} = 150$, then the optimal solution under ROI maximization solved by SAS is $Q_f^* = 17.36$ and the corresponding unique global optimal ROI is 5.4312. When there is an option to invest additional money in setup operations and/or quality improvement, the optimal solutions obtained by SAS are $Q^* = 3.83$, $K_s^* = 109.66$,

Table 1 Sensitivity analysis for C.

Unit Variable Cost	Q^*_{UG}	K^*_{sUG}	K^*_{rUG}	R^*_{UG}
100	3.831	109.66	307.12	12.252
102	3.737	110.91	312.58	12.049
104	3.646	112.16	318.03	11.852
106	3.560	113.39	323.48	11.663
108	3.477	114.61	328.94	11.479

and $K^*_r = 307.12$, and the corresponding unique global optimal ROI is 12.2524. Since $K^*_{sUG} + K^*_{rUG} = 416.78 < K_{max} = 500$, the capital budget constraint is also satisfied in this case. We note that when we invest additional money in both setup cost reduction and quality improvement, the order quantity is reduced and ROI increases.

It is interesting to investigate some sensitivity analysis, especially unit variable cost, which is summarized in Table 1. It is interesting to note that when variable unit cost increases, the investments in both setup operations and quality improvement increase, but the order quantity and the level of ROI decreases.

5. Concluding Remarks

In this paper, we constructed and analyzed inventory and capital investment allocation policies under return on investment (ROI) maximization. Our model was constructed for a decision maker of a single product with a budget constraint in capital investment. First, we showed how to formulate ROI maximization problems. Under Problem A, a decision maker of an inventory system with a single product does not have an option to invest additional money in setup cost reduction and/or quality improvement. On the other hand, under Problem B, a decision maker has an option to invest

additional money in setup cost reduction and/or quality improvement.

Moreover, we showed how the levels for the prior and posterior order quantities are reduced when it is optimal to invest additional money in setup cost reduction and/or quality improvement. In addition, the unique global optimal solution is determined by employing the primary criterion of ROI maximization, the secondary criterion of the posterior order quantity minimization (i.e., inventory reduction), and the tertiary criterion of the prior order quantity minimization.

Furthermore, for the specific case of a rational setup cost function with a linear quality improvement function, we illustrated a numerical example to show sensitivity analysis of unit variable cost.

There are several extensions that will further enhance the importance and relevance of our model. For example, in our model, we assumed a single product. If we consider several products that have economic relations (i.e., substitutes and complements; see Chapter 5 and Chapter 6), the formulation and analyses must be adjusted accordingly (e.g., how to allocate capital investments for substitute product. Also, if we relax the assumption of zero cost/value of defective items, it would be of interest to analyze various scenarios such as rework and/or salvage value of the defective items.

References

Billington, Peter. J. (1987) The classical economic production quantity model with setup cost as a function of capital expenditure. *Decision Sciences*, *18*, 25–42.

Cheng, T.C.E. (1991). EPQ with process capability and quality assurance considerations," *Journal of Operations Research Society*, *42*(8), 713–720.

Hillier, Frederick S. and Lieberman, Gerald J. (1995). *Introduction to operations research*, sixth edition, New York, New York: McGraw-Hill Publishing Company.

Hong, Jae-Dong, Xu, S. H., and Hayya, J. C. (1993). Process quality improvement and setup reduction in dynamic lot-sizing. *International Journal of Production Research*, *33*(11), 2693–2708.

Hong, Jae-Dong, and Hayya, Jack C. (1995). Joint investment in quality improvement and setup reduction. *Computers and Operations Research*, *22*(6), 567–574.

Kekre, Sunder and Mukhopadhyay, Tridas (1992). Impact of electronic data interchange technology on quality improvement and inventory reduction programs: A field study. *International Journal of Production Economics*, *28*, 265–282.

Kim, Seung Lae, Hayya, Jack C., and Hong, Jae-Dong (1992). Setup reduction in the economic production quantity model. *Decision Sciences*, *23*, 500–508.

Ladany, S. and Sternlieb, A. (1974) The interaction of economic ordering quantities and marketing policies. *AIIE Transactions*, *6*(1), 35–40.

Lee, Hau L. and Rosenblatt, Meir J. (1985). Optimal inspection and ordering policies for products with imperfect quality. *IIE Transactions*, *17*(3), 284–289.

Leschke, John P. (1997a). The setup-reduction process: Part 1. *Production and Inventory Management Journal*, 32–37.

Leschke, John P. (1997b). The setup-reduction process: Part 2–setting reduction priorities. *Production and Inventory Management Journal*, 38–42.

Leschke, John P. and Weiss, Elliott N. (1997). The multi-item setup-reduction investment-allocation problem with continuous investment-cost functions. *Management Science*, *43*(6), 890–89.

Luenberger, David G. (1984). *Linear and nonlinear programming*, second edition, Reading, MA: Addison-Wesley Publishing Company.

Morse, Wayne J. and Scheiner, James H. (1979). Cost minimization, return on investment, residual income: alternative criteria for inventory models," *Accounting and Business Research*, *9*(3), 320–324.

Oakleaf, Robert B. (1972). Retail trade ROI. *Management Accounting*, 25–26.

Porteus, Evan L. (1985). Investing in reduced setups in the EOQ model. *Management Science*, *31*(8), 998–1010.

Porteus, Evan L. (1986). Investing in new parameter values in the discounted EOQ model. *Naval Research Logistics Quarterly*, *33*, 39–48.

Reece, James S and Cool, William R. (1978). Measuring investment center performance. *Harvard Business Review*, *56*(3), 28–180.

Rosenberg, David. (1991). Optimal price-inventory decisions: profit vs. ROII. *IIE Transactions*, *23*(1), 17–22.

Rosenblatt, Meir J., and Rothblum, Uriel G. (1994). The optimality of the "cut across the board" rule applied to an inventory model. *IIE Transaction*, *26*(2), 102–108.

SAS Institute Inc. (1995). *SAS/IML Sofrware: Changes and Enhancements.* Cary, NC: SAS Institute Inc.

Schroeder, Roger G. and Krishnan, Ramakrishnan (1976). Return on investment as a criterion for inventory models. *Decision Sciences*, *7*(4), 697–704.

Smith, W. M. (1958). An investigation of some quantitative relationships between breakeven point analysis and economic lot size theory. *AIIE Transactions*, *9*, 52–57.

Voss, C.A. (1987). *International trends in manufacturing technology: Just-In-Time manufacture.* Linz, Austria: IFS Publications.

Whitin, T. M. (1955). Inventory control and price theory. *Management Science*, *2*, 61–68.

CHAPTER 4. INVENTORY AND INVESTMENT IN SETUP OPERATIONS UNDER PROFIT AND ROI MAXIMIZATION

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Industrial Engineering Research Conference [1]

Toshitsugu Otake and K. Jo Min

Abstract

We investigate inventory and investment in setup operations policies under a profit maximization model and a return on investment (ROI) maximization model. We examine the optimality conditions for both models and study how inventory is reduced when it is optimal to invest additional money in setup operations. Furthermore, we compare and contrast the inventory reduction between the profit model and the ROI model. We also examine the unique global optimal solutions in closed-form when the setup cost is a rational or linear function of the level of investment. Finally, we illustrate various interesting observations on our models via numerical examples.

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Keywords

Economic Order Quantity, Inventory Reduction, Profit, ROI, and Setup Operations.

1. Introduction

In this paper, we investigate inventory and investment in setup operations policies under profit maximization and return on investment (ROI) maximization for a decision maker with single product of inventory systems.

In the literature of inventory control, numerous papers have employed the profit maximization (or cost minimization) as their objective in designing and analyzing inventory models (see e.g., Whitin 1955; Smith 1958; Ladany and Sternlieb 1974; Hillier and Lieberman 1995). ROI is also a widely utilized economic and finance performance measure dealing with finished goods inventories (see e.g., Schroeder and Krishnan 1976; Morse and Scheiner 1979; Reece and Cool 1978).

Thus far, the inventory literature on performance criteria have been reviewed. Let us now review investment in setup operations. Porteus (1985) pointed out that Japanese devoted to decreasing setup cost in their manufacturing processes and he provided an undiscounted EOQ model. Furthermore, Porteus (1986) extended Porteus (1985) to the case of discounted EOQ model. Billington (1987) formulates a model of which setup cost is a function of capital expenses and investigates the relations among holding, setup, and capital expenses. Hong, Xu, and Hayya (1993) proposes a dynamic lot-sizing model of which setup reduction and process quality are functions of capital expenditure. Kim, Hayya, and Hong (1992) investigates several classes of setup reduction functions by

employing the economic production quantity model.

The rest of this paper is organized as follows: we first formulate a profit maximization model and an ROI maximization model. Then, we examine the characteristics of solutions under profit maximization and ROI maximization. Moreover, by employing rational and linear setup cost functions, we obtain the unique global optimal solutions in closed-form. Also several interesting managerial insights are provided. Finally, concluding remarks are presented.

2. Optimality Conditions

2.1 Definitions and assumptions

First of all, for a decision maker with a single product, various notations and definitions used throughout this paper are as follows:

Q: the order quantity.

C: the variable cost per unit.

I: the inventory holding cost expressed as a fraction of the unit cost per unit time, which includes the opportunity cost of funds tied up in inventory.

i: the inventory holding cost expressed as a fraction of the unit cost per unit time, which excludes the opportunity cost of funds tied up in inventory.

K: the capital investment per unit time in setup operation.

$S(K)$: the setup cost as a function of K.

P: the selling price per unit.

D: the sales quantity per unit time.

In addition, we utilize assumptions of EOQ-type papers such as no shortage and no delivery lag (see Morse and Scheiner 1979).

2.2 Profit maximization model

For the profit maximization model, we consider two types of the profit maximization problem as Problem A and Problem B. Under Problem A, profit is maximized over Q given the current level of the investment in equipment and training, K_F. The total cost per unit time, TC, consists of costs of the setup cost, the variable cost, and the holding cost and the capital investment per unit time in setup operations (see e.g., Billington 1987). Hence, mathematically, $TC = \frac{S_F D}{Q} + CD + \frac{ICQ}{2} + K_F$ where $S_F = S(K_F)$.

Since the total revenue per unit time is the selling price per unit multiplied by the sales quantity per unit time (i.e., PD), the profit per unit time, Π, is obtained by subtracting the total cost per unit time from the revenue per unit time. Since profit is maximized over the order quantity, an equivalent model formulation (see Luenberger 1984) as in Chen (1995) for Problem A is given by

$$\textbf{Problem A:} \quad \min_{Q} \quad -\Pi_F = -PD + \frac{S_F D}{Q} + CD + \frac{ICQ}{2} + K_F \tag{1}$$

From the first order necessary condition (FONC), we obtain the following optimal solution for Problem A:

$$Q^*_{\Pi_F} = \sqrt{\frac{2S_F D}{IC}} \tag{2}$$

Since the second order sufficient condition is satisfied at optimality, $Q^*_{\Pi_F}$ is the unique global solution for Problem A and the corresponding profit, Π^*_F, is the global optimal profit.

On the other hand, for Problem B, the decision maker has an option to invest additional money in setup operations. Hence, under Problem B, Π is maximized over Q as well as K for $K_{min} \leq K \leq K_{max}$ where K_{min} represents the current level of K, K_F, while K_{max} represents the technologically feasible maximum investment. We will denote the corresponding setup costs $S(K_{min})$ and $S(K_{max})$ as S_{min} and S_{max}, respectively.

$$\textbf{Problem B:} \quad \min_{Q,\,K} \quad -\Pi = -PD + \frac{S(K)D}{Q} + CD + \frac{ICQ}{2} + K \tag{3}$$

$$\text{subject to } K_{min} - K \leq 0 \text{ and } K - K_{max} \leq 0.$$

Under Problem B, there are three possible cases to be considered. Optimal solutions for $K^* = K_{min}$, $K^* = K_{max}$, and $K^* = K^*_{\Pi_{int}} \in (K_{min}, K_{max})$, are $Q^*_{\Pi_{min}}$, $Q^*_{\Pi_{max}}$, and $Q^*_{\Pi_{int}}$, respectively. Similarly, optimal objective function value for $K^* = K_{min}$, $K^* = K_{max}$, and $K^* = K^*_{\Pi_{int}}$, are expressed as Π^*_{min}, Π^*_{max}, and Π^*_{int}, respectively. We will assume that the Second Order Sufficient Conditions (SOSC) are satisfied. Especially, for an interior local optimal solution, the corresponding SOSC is expressed by

$$2S^*_{\Pi_{int}} S^{*\prime\prime}_{\Pi_{int}} > (S^{*\prime}_{\Pi_{int}})^2 \tag{4}$$

2.3 ROI maximization model

Similar to the previous subsection, we consider two types of the ROI maximization problem as Problem C and Problem D. Under Problem C, ROI is maximized over Q given the current level of the investment in equipment and training, K_F. The inventory has been widely viewed as a capital investment for profits (see Schroeder and Krishnan 1976; Morse and Scheiner 1979; Oakleaf 1972) and the capital investment in setup operations is also viewed as an investment. Hence, the average investment per unit time is given by $\frac{CQ}{2} + K_F$. Since ROI is expressed as the ratio of the profit per unit time over the average investment per unit time, ROI given K_F as in Chen (1995) is obtained as follows:

$$R_F = (PD - \frac{S_F D}{Q} - CD - \frac{iCQ}{2} - K_F)/(\frac{CQ}{2} + K_F) \tag{5}$$

Since ROI is maximized over the order quantity, an equivalent model formulation (see Luenberger 1984) for Problem C is given by

$$\textbf{Problem C:} \qquad \min_{Q} \; -R_F \tag{6}$$

From the first order necessary condition (FONC), we obtain the following equation:

$$Q^*_{R_F} = [CDS_F + (2CDK_F S_F \mathcal{M}_F + C^2 D^2 S_F^2)^{0.5}]/(C\mathcal{M}_F) \tag{7}$$

where $\mathcal{M}_F = PD - CD - K_F + iK_F$. It can be verified that $Q^*_{R_F}$ is unique and satisfies the second order sufficient condition (SOSC). Hence, $Q^*_{R_F}$ is the unique global optimal

solution for Problem A and the corresponding ROI, R_F^*, is the global optimal ROI.

Now, under Problem D, ROI is maximized over Q as well as K for $K_{min} \leq K \leq K_{max}$. An equivalent model formulation for Problem D is given below.

$$\textbf{Problem D:} \quad \min_{Q,\,K} \quad -R = \left(\frac{S(K)D}{Q} + CD + \frac{iCQ}{2} + K - PD\right)/\left(\frac{CQ}{2} + K\right) \tag{8}$$

$$\text{subject to } K_{min} - K \leq 0 \text{ and } K - K_{max} \leq 0.$$

Under Problem D, there are three possible cases to be considered. Optimal solutions for $K^* = K_{min}$, $K^* = K_{max}$, and $K^*_{R_{int}} \in (K_{min}, K_{max})$, are $Q^*_{R_{min}}$, $Q^*_{R_{max}}$, and $Q^*_{R_{int}}$, respectively. Similarly, optimal objective function value for $K^* = K_{min}$, $K^* = K_{max}$, $K^* = K^*_{R_{int}}$, are expressed as R^*_{min}, R^*_{max}, and R^*_{int}, respectively. We will assume that SOSC is satisfied. Especially, for an interior local optimal solution, the corresponding SOSC is expressed by

$$2S^*_{R_{int}} S^{*\prime\prime}_{R_{int}} > (S^{*\prime}_{R_{int}})^2 \tag{9}$$

We note that the detailed design and analysis of the ROI model are summarized in Otake, Chen, and Min (1997).

3. Analysis under a General Setup Cost Function

3.1 Inventory reduction analysis

In this subsection, we will examine if the option to invest in setup operations results in inventory reduction. Let us now suppose that there are m ($m \geq 1$) interior local optimal solutions designated by $(Q^{i*}_{\Pi_{int}}, K^{i*}_{\Pi_{int}})$, $i = 1, \ldots, m$. We denote the corresponding profits by Π^{i*}_{int}, $i = 1, \ldots, m$. Let us denote a global optimal solution for Problem B and Problem D by $(Q^*_{\Pi_G}, K^*_{\Pi_G})$ and $(Q^*_{R_G}, K^*_{R_G})$, and the corresponding profit and ROI by Π^*_G and R^*_G, respectively. In order to show this, first, we will compare the global optimal order quantity for Problem A, $Q^*_{\Pi_F}$, with that for Problem B, $Q^*_{\Pi_G}$. Based on the FONCs, we can have the following Proposition 1:

Proposition 1. (Inventory Reduction for Profit Maximization Problem)

1) If $\Pi^*_G = \Pi^*_{max}$, then the level of inventory is reduced, and the reduction in the order quantity is given by $Q^*_{\Pi_F} - Q^*_{\Pi_{max}}$.

2) If $\Pi^*_G = \Pi^{i*}_{int}$, then the level of inventory is reduced, and the reduction in the order quantity is $Q^*_{\Pi_F} - Q^{i*}_{\Pi_{int}}$.

3) If $\Pi^*_G = \Pi^*_{min}$, then the level of inventory remains the same, and the reduction in the order quantity is zero.

PROOF: Proof is similar to that in Proposition 1 of Chapter 2. ■

Similarly, we note that detailed investigation for Proposition 2 for the case of ROI model is summarized in Otake, Chen, and Min (1997).

Proposition 2. (Inventory Reduction for ROI Maximization Problem)

1) If $R_G^* = R_{max}^*$, then the level of inventory is reduced, and the reduction in the order quantity is given by $Q_{R_F}^* - Q_{R_{max}}^*$.

2) If $R_G^* = R_{int}^{i*}$, then the level of inventory is reduced, and the reduction in the order quantity is $Q_{R_F}^* - Q_{R_{int}}^{i*}$.

3) If $R_G^* = R_{min}^*$, then the level of inventory remains the same, and the reduction in the order quantity is zero.

PROOF: Proof is similar to that in Proposition 1 of Chapter 2. ■

Hence, if the decision maker finds it optimal to invest additional money in setup operations, the level of inventory will be always reduced under both the profit maximization model and the ROI maximization model.

3.2 Derivation of unique global optimal solutions

In this subsection, we will employ an additional criterion based on Proposition 1 and Proposition 2 to induce a unique global optimal solution. The additional criterion is: if the levels of ROI are the same, then the global optimal solution with the lowest level of order quantity will be preferred. Given that the levels of financial performance are the same, the smallest inventory is the most preferable due to factors that are external to this model (e.g., storage facilities, space, risk of deterioration and obsoleteness, etc.).

In our model formulation for profit maximization, given the multiple global optimal solutions with Π_G^*, we can show that the smallest order quantity is associated with the largest capital investment. Similarly, for ROI maximization, given the multiple global

optimal solutions with R^*_G, we can show that the smallest order quantity is associated with the largest capital investment.

Hence, if there are more than one global optimal solutions under profit maximization or ROI maximization as the primary criterion, then the global optimal solution with the largest capital investment will be the unique global optimal solution under the order quantity minimization as the secondary criterion.

3.3 Comparison of unique global inventory level

Now, in this subsection, we compare and contrast the unique global optimal solution for profit maximization with that for ROI maximization. Comparison of unique global inventory levels under profit maximization and ROI maximization is shown. We note that the level of the unique global order quantity indicates the level of unique global optimal inventory.

Suppose that the opportunity cost of funds tied up in inventory is less than or equal to the optimal ROI level (i.e., $I - i \leq R^*$).

1) if $K^*_{UG} = K_{min}$ under the profit maximization model, then $Q^*_{R_{UG}} \leq Q^*_{\Pi_{UG}}$.

2) if $K^*_{UG} = K^{i*}_{\Pi_{int}}$ under the profit maximization model, then we have the following two cases.

a) and if $K^{i*}_{\Pi_{int}} \leq K^*_{UG} = K^{j*}_{R_{int}} \leq K_{max}$ under the ROI maximization model, then $Q^*_{R_{UG}} \leq Q^*_{\Pi_{UG}}$.

b) otherwise, the relation between $Q^*_{R_{UG}}$ and $Q^*_{\Pi_{UG}}$ is undetermined.

3) if $K^*_{UG} = K_{max}$ under the profit maximization model, then we have the following two

cases.

a) and if $K^*_{UG} = K_{max}$ under the ROI maximization model, then $Q^*_{R_{UG}} \leq Q^*_{\Pi_{UG}}$.

b) otherwise, the relation between $Q^*_{R_{UG}}$ and $Q^*_{\Pi_{UG}}$ is undetermined.

Suppose that the opportunity cost of funds tied up in inventory is greater than the optimal ROI level (i.e., $I - i > R^*$). Then, the relation between $Q^*_{R_{UG}}$ and $Q^*_{\Pi_{UG}}$ is undetermined.

4. Analysis under a Rational Setup Cost Function

In this section, we consider a rational function as a setup cost function, $S(K) = \frac{\gamma}{K}$ where γ is a positive constant, in order to show additional managerial insights. This function is characterized as constant elasticity over any level of investment (see Chen 1995). Furthermore, it can be easily verified that, for the rational setup cost function under profit maximization, both the boundary solutions and the interior solutions can be optimal. Then, we have the following Proposition 3 as in Chen (1995):

Proposition 3. (Decision Making Rules for Rational Setup Cost Function under Profit Maximization)

If $K_{min} < K^*_{\Pi_{int}} = \sqrt[3]{\frac{IC\gamma D}{2}} < K_{max}$, then $K^*_{\Pi_{UG}} = \sqrt[3]{\frac{IC\gamma D}{2}}$ and $Q^*_{\Pi_{UG}} = \sqrt[3]{\frac{4\gamma D}{I^2C^2}}$.

Otherwise,

1) if $IC < \frac{K_{max}-K_{min}}{Q^*_{\Pi_{min}}-Q^*_{\Pi_{max}}}$, then $K^*_{\Pi_{UG}} = K_{min}$ and $Q^*_{\Pi_{UG}} = \sqrt{\frac{2\gamma D}{K_{min}IC}}$.

2) if $IC > \frac{K_{max}-K_{min}}{Q^*_{\Pi_{min}}-Q^*_{\Pi_{max}}}$, then $K^*_{\Pi_{UG}} = K_{max}$ and $Q^*_{\Pi_{UG}} = \sqrt{\frac{2\gamma D}{K_{max}IC}}$.

When some local optimal cases do not exist (e.g., $K^*_{\Pi_{int}}$ is never optimal), similar decision making rules are also provided. The subsequent analyses are simpler because

of the absence of some local optimal solutions. For decision making rules for rational setup cost function under ROI maximization, details are shown in Otake, Chen, and Min (1997). We now proceed to illustrate a numerical example below.

Example 1

Let us suppose that $C = \$100$, $D = 25$ per month, $I = 0.2$ per month, $i = 0.1$ per month, $P = \$150$, $K_{min} = \$50$ per month, $K_{max} = \$480$ per month, and $\gamma = 15000$. We note that these numerical values are identical to these in Chen (1995). However, the numerical example is different here. Our emphasis is on inventory reduction, which was NOT addressed in Chen (1995). When initial investment level is fixed at $K_F = 50$, the unique global optimal solutions under profit maximization and ROI maximization are $(Q^*_{\Pi_F}, K_{\Pi_F}) = (27.39, 50)$ and $(Q^*_{R_F}, K_{R_F}) = (12.93, 50)$ and the corresponding unique global optimal profit and ROI are 652 and 0.80, respectively. On the other hand, when there is an option to invest additional money in setup operations, we obtain the unique global optimal solutions under profit maximization and ROI maximization as $(Q^*_{\Pi_{UG}}, K^*_{\Pi_{UG}}) = (15.54, 155.36)$ and $(Q^*_{R_{UG}}, K^*_{R_{UG}}) = (5.33, 169.05)$, respectively. Also, the corresponding unique global optimal profit and ROI are 784 and 1.47, respectively. We note that when there is an option to invest additional money in setup operations, profit is improved from 652 to 784 and level of inventory is reduced from 27.39 to 15.54. Similarly, when there is an option to invest additional money in setup operations, ROI is improved by 0.67 and the level of inventory is reduced by 7.60. Furthermore, since the unique global optimal ROI is greater than the opportunity cost of funds tied up in inventory and level of investment under ROI maximization is greater than that under

profit maximization, we note that level of inventory under ROI maximization, 5.33, is less than that under profit maximization, 15.54.

5. Analysis under a Linear Setup Cost Function

In this section, we consider a linear function as a setup cost function, $S(K) = \alpha - \beta K$ where α and β are positive constants. Contrary to the rational setup cost function, it is interesting to note that there does not exist interior local optimal solutions under the linear setup cost function. Then, we have the following Proposition 4 as in Chen (1995):

Proposition 4. (Decision Making Rules for Linear Setup Cost Function under Profit Maximization)

1) If $IC < \frac{K_{max}-K_{min}}{Q^*_{\Pi_{min}}-Q^*_{\Pi_{max}}}$, then $K^*_{\Pi_{UG}} = K_{min}$ and $Q^*_{\Pi_{UG}} = \sqrt{\frac{2(\alpha-\beta K_{min})D}{IC}}$.

2) If $IC > \frac{K_{max}-K_{min}}{Q^*_{\Pi_{min}}-Q^*_{\Pi_{max}}}$, then $K^*_{\Pi_{UG}} = K_{max}$ and $Q^*_{\Pi_{UG}} = \sqrt{\frac{2(\alpha-\beta K_{max})D}{IC}}$.

When some local optimal cases do not exist (e.g., K_{min} is never optimal), similar decision making rules are also provided. The subsequent analyses are simpler because of the absence of some local optimal solutions. For decision making rules for linear setup cost function under ROI maximization, details are shown in Otake, Chen, and Min (1997). We again proceed to illustrate a numerical example below.

Example 2

Let us suppose that C = \$100, D = 25 per month, I = 0.2 per month, i = 0.1 per month, P = \$150, K_{min} = \$50, K_{max} = \$480, α = 500, and β = 1. We note that these numerical values are identical to these in Chen (1995). However, the numerical example is different here. Our emphasis is on inventory reduction, which was NOT

addressed in Chen (1995). When initial investment level is fixed at $K_F = 50$, the unique global optimal solutions under profit maximization and ROI maximization are $(Q^*_{\Pi_F}, K_{\Pi_F}) = (33.37, 50)$ and $(Q^*_{R_F}, K_{R_F}) = (19.5, 50)$ and the corresponding profit and ROI are 501 and 0.50, respectively. On the other hand, when there is an option to invest additional money in setup operations, we obtain the unique global optimal solutions under profit maximization and ROI maximization as $(Q^*_{\Pi_{UG}}, K^*_{\Pi_{UG}}) = (7.04, 480)$ and $(Q^*_{R_{UG}}, K^*_{R_{UG}}) = (3.11, 480)$, respectively. Also, the corresponding unique global optimal profit and ROI are 603 and 0.934, respectively. We note that when there is an option to invest additional money in setup operations, both profit and ROI are improved and level of inventory under both cases is reduced. Furthermore, even if the level of the investment under profit and ROI maximization is the same as the maximum investment level, we note that level of inventory under ROI maximization is less than that under profit maximization.

6. Concluding Remarks

In this paper, we investigated inventory policies and investment in setup operations policies under profit maximization and ROI maximization. First, we studied how a profit maximization problem and an ROI maximization problem are formulated. Second, we examined the unique global optimal solution by the primary criterion of profit maximization or ROI maximization and the secondary criterion of the order quantity minimization.

In addition, we studied how the level of order quantities(i.e., the level of inventory)

under profit and ROI maximization are reduced when it is optimal to invest additional money in setup operations. Furthermore, by employing the secondary criterion, we compared and contrasted the unique global optimal solutions under profit and ROI maximization.

Finally, under the assumption of rational and linear setup cost functions, we first obtained the unique global optimal solutions and provided the decision making rules to determine the unique global optimal solution.

There are several extensions that will further enhance the importance and relevance of our model. They include incorporation of more sophisticated features such as shortages, delivery lags, and stochastic demand rates, etc. Also, it would be of interest to study the allocation of the investment in setup operations and quality improvement incorporating stochastic nature.

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References

Billington, Peter. J. (1987). The classical economic production quantity model with setup cost as a function of capital expenditure. *Decision Sciences, 18*, 25–42.

Chen, Cheng-Kang (1995). *Market-based allocation mechanisms for lot-size decision makers and electric power utilities* (Ph.D. dissertation). Ames, Iowa: Iowa State University.

Hillier, Frederick S. and Lieberman, Gerald J. (1995). *Introduction to operations research*, sixth edition. New York, New York: McGraw-Hill Publishing Company.

Hong, Jae-Dong, Xu, S. H., and Hayya, J. C. (1993). Process quality improvement and setup reduction in dynamic lot-sizing. *International Journal of Production Research*, *33*(11), 2693–2708.

Kim, Seung Lae, Hayya, Jack C., and Hong, Jae-Dong. (1992). Setup reduction in the economic production quantity model. *Decision Sciences*, *23*, 500–508.

Ladany, S. and Sternlieb, A. (1974). The Interaction of economic ordering quantities and marketing policies. *AIIE Transactions*, *6* (1), 35–40.

Luenberger, David G. (1984). *Linear and nonlinear programming*, second edition. Reading, MA: Addison-Wesley Publishing Company.

Morse, Wayne J and Scheiner, James H. (1979). Cost minimization, return on investment, residual income: alternative criteria for inventory models. *Accounting and Business Research*, *9*(3), 320–324.

Oakleaf, Robert B. (1972). Retail trade ROI. *Management Accounting*, 25–26.

Otake, Toshitsugu, Chen, Cheng-Kang, and Min, K. Jo. (1997). Inventory and investment in setup operations under ROI maximization. *Working Paper*, Ames, IA: IMSE Department, Iowa State University.

Porteus, Evan L. (1985). Investing in reduced setups in the EOQ model. *Management Science*, **31** (8), 998–1010.

Porteus, Evan L. (1986). Investing in new parameter values in the discounted EOQ model. *Naval Research Logistics Quarterly*, **33**, 39–48.

Reece, James S and Cool, William R. (1978). Measuring investment center performance. *Harvard Business Review*, **56** (3), 28–180.

Schroeder, Roger G. and Krishnan, Ramakrishnan (1976). Return on investment as a criterion for inventory models. *Decision Sciences*, *7*(4), 697–704.

Smith, W. M. (1958). An investigation of some quantitative relationships between breakeven point analysis and economic lot size theory. *AIIE Transactions*, *9*, 52–57.

Whitin, T. M. (1955). Inventory control and price theory. *Management Science*, *2*, 61–68.

CHAPTER 5. INVENTORY AND PRICING POLICIES FOR A DUOPOLY OF SUBSTITUTE PRODUCTS

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Industrial Engineering Research Conference [1]

Toshitsugu Otake and K. Jo Min

Abstract

We design and analyze two duopoly models for two profit maximizing sellers. Each seller is assumed to produce one product, and his competitor is assumed to produce a substitute. Under the behavioral assumptions of a Cournot-type model and a Bertrand-type model, we derive the equilibrium conditions for both models given linear demand and inverse demand functions. Next, under the assumption of symmetric costs, we derive the closed form inventory and pricing policies at equilibrium. Numerous interesting economic implications are obtained via calculus and numerical analyses.

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Keywords

Economic Order Quantity, Duopoly, Substitutes, and Pricing.

1. Introduction

Recently, there have been numerous papers investigating inventory and pricing policies under competition (see, e.g., Min 1992; Chen and Min 1995). There are few papers, however, that investigate the impacts of substitutes on the inventory and pricing policies. Given the prevalence of substitute products in real world inventory and pricing policies, it is highly desirable to explore the impacts of substitutes.

As a first step toward the full exploration, in this paper, we design and analyze competitive inventory models for *two* sellers. We assume that each seller produces a single product that can be a substitute for the competing seller's product. The quantitative relations among the substitutes and their corresponding prices are expressed by the demand functions of the two products. In characterizing the competitive behavior of each seller, we employ a Cournot-type model and a Bertrand-type model (see. e.g., Varian 1992).

Under the Cournot-type competitive model, we assume that each seller maximizes his profit per unit time over his order quantity and his demand (i.e., sale) per unit time assuming a given level of demand (i.e., sale) per unit time of his competitor. On the other hand, under the Bertrand-type competitive behavior, we assume that each seller maximizes his profit per unit time over his order quantity and his price per unit assuming a given level of price per unit of his competitor. We note that for both the Cournot-type

model and the Bertrand-type model, the determination of prices imply the determination of demands and vice versa. Hence, the pricing policies necessarily determine the sale (measured in demand per unit time) policies and vice versa. For both the Cournot-type model and the Bertrand-type model, we will assume that demands are linear functions of prices (and vice versa). This linearity assumption can be found in numerous papers and books (see, e.g., Choi 1991) and facilitates the analyses of our models.

2. Two Types of Basic Models

2.1 Definitions and assumptions

For the Cournot-type model, we employ the linear inverse demand function as follows.

$$P_1 = \alpha - \beta d_1 - \gamma d_2 \quad (1)$$

$$P_2 = \alpha - \gamma d_1 - \beta d_2 \quad (2)$$

where

P_i: the per unit price of product i, i=1,2

d_i: the per unit time demand of product i, i=1,2

α: the intercept of the inverse demand function

β: the own price effect

γ: the cross price effect

and parameters, α, β, and γ, are positive. The cross price effects are symmetric as is required for well-behaved consumer demand function (Varian 1992). Also, $\beta > \gamma$ and the difference $\beta - \gamma$ is directly related to the degree of product substitutability between the two products (Choi 1991).

Similarly, for the Bertrand-type model, we employ the direct linear demand function as follows.

$$d_1 = a - bP_1 + \delta P_2 \tag{3}$$

$$d_2 = a + \delta P_1 - bP_2 \tag{4}$$

where

a: the intercept of the demand function

b: the own demand effect

δ: the cross demand effect

where parameters, a, b, and δ, are positive and the cross demand effects are symmetric and b > δ and the difference b - δ is inversely related to the degree of product substitutability between the two products.

In order to mathematically formulate these models, the following variables and parameters are defined: For i = 1, 2,

Q_i: the order quantity of product i for seller i

A_i: the set up cost of product i

C_i: the variable cost per unit time of product i

H_i: the inventory holding cost per unit per unit time of product i

T_i: the cycle length for product i.

The basic assumptions for traditional EOQ model applied in this paper are as follows:

1) buyer's demand rate is constant over time,

2) the replenishment rate is infinite,

3) no shortage is allowed,

4) there is no delivery lag.

2.2 Cournot-type model with linear demand

Under the definitions and assumptions shown above, we design and analyze the Cournot-type model as follows. For Seller 1, per unit time profit maximization problem is:

$$\begin{aligned} \max_{Q_1,d_1} \Pi_1(Q_1,d_1|\bar{d}_2) &= (\alpha-\beta d_1-\gamma\bar{d}_2)d_1 \\ &- \frac{A_1 d_1}{Q_1} - C_1 d_1 - \frac{1}{2}H_1 Q_1 \end{aligned} \tag{5}$$

where $\bar{d}_2$ denotes a given level of demand per unit time for Seller 2.

Similarly, for Seller 2,

$$\max_{Q_2,d_2} \Pi_2(Q_2, d_2|\bar{d}_1) = (\alpha - \gamma\bar{d}_1 - \beta d_2)d_2 - \frac{A_2 d_2}{Q_2} - C_2 d_2 - \frac{1}{2}H_2 Q_2 \quad (6)$$

where $\bar{d}_1$ denotes a given level of demand per unit time for Seller 1.

The corresponding first order necessary conditions (FONC) for (5) are:

$$\frac{\partial \Pi_1}{\partial d_1} = \alpha - 2\beta d_1 - \gamma\bar{d}_2 - \frac{A_1}{Q_1} - C_1 = 0 \quad (7)$$

$$\frac{\partial \Pi_1}{\partial Q_1} = \frac{A_1 d_1}{Q_1^2} - \frac{1}{2}H_1 = 0 \quad (8)$$

Meanwhile, the corresponding first order necessary conditions (FONC) for (6) are:

$$\frac{\partial \Pi_2}{\partial d_2} = \alpha - 2\beta d_2 - \gamma\bar{d}_1 - \frac{A_2}{Q_2} - C_2 = 0 \quad (9)$$

$$\frac{\partial \Pi_2}{\partial Q_2} = \frac{A_2 d_2}{Q_2^2} - \frac{1}{2}H_2 = 0 \quad (10)$$

From the cubic equation formula in the Standard Mathematical Table (Beyer 1981), we have the following trigonometric form for d_1 and Q_1.

$$d_1 = \frac{2(\alpha - \gamma\bar{d}_2 - C_1)}{3\beta}\cos^2\frac{\theta_1}{3} \quad (11)$$

$$Q_1 = [\frac{4A_1(\alpha - \gamma\bar{d}_2 - C_1)}{3\beta H_1}]^{\frac{1}{2}} \cos\frac{\theta_1}{3} \tag{12}$$

where

$$\cos\theta_1 = -[\frac{27\beta H_1 A_1}{4(\alpha - \gamma\bar{d}_2 - C_1)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_1 \leq \frac{3}{4}\pi$ assuming non-negative profit (see, e.g., Chen and Min 1994).

The corresponding second order sufficient condition (SOSC) is given by $4Q_1 d_1\beta - A_1 > 0$. Similarly, the decision variables for Seller 2 are also obtained as

$$d_2 = \frac{2(\alpha - \gamma\bar{d}_1 - C_2)}{3\beta}\cos^2\frac{\theta_2}{3} \tag{13}$$

$$Q_2 = [\frac{4A_2(\alpha - \gamma\bar{d}_1 - C_2)}{3\beta H_2}]^{\frac{1}{2}} \cos\frac{\theta_2}{3} \tag{14}$$

where

$$\cos\theta_2 = -[\frac{27\beta H_2 A_2}{4(\alpha - \gamma\bar{d}_1 - C_2)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_2 \leq \frac{3}{4}\pi$.

The corresponding second order sufficient condition (SOSC) is given by $4Q_2 d_2\beta - A_2 > 0$.

At an equilibrium point, $d_1 = \bar{d}_1$ and $d_2 = \bar{d}_2$. Hence, the equilibrium point can be obtained by solving (11), (12), (13), (14) given $d_1 = \bar{d}_1$ and $d_2 = \bar{d}_2$ for d_1^*, d_2^*, Q_1^*, and Q_2^*. For the basic Cournot-type model, it has not been possible to obtain a closed form

equilibrium point. Therefore, we employ numerical methods to solve for the equilibrium point.

Example 1.

Let A_1 = \$1000, H_1 = \$4.0, C_1 = \$20, A_2 = \$750, H_2 = \$3.0, C_2 = \$15, α=100, β=1.0, and γ=0.5. Then, the corresponding equilibrium point is given by (d_1^*, Q_1^*)=(27.6, 117.4), (d_2^*, Q_2^*)=(32.7, 127.8), and Π_1^*=525, and Π_2^*=875. Given the same price and cross price effects, the profit difference can be explained by the cost difference.

2.3 Bertrand-type model with linear demand

As mentioned before, the Bertrand-type model involves the price levels instead of the demand levels as the decision variables. Thus, for Seller 1, per unit time profit maximization problem for the Bertrand-type model is derived as follows.

$$\begin{aligned} \max_{Q_1, P_1} \Pi_1(Q_1, P_1 | \bar{P}_2) &= P_1(a - bP_1 + \delta \bar{P}_2) \\ &- \frac{A_1(a - bP_1 + \delta \bar{P}_2)}{Q_1} \\ &- C_1(a - bP_1 + \delta \bar{P}_2) \\ &- \frac{1}{2} h_1 Q_1 \end{aligned} \tag{15}$$

where $\bar{P}_2$ denotes a given level of per unit price for product 2.

Similarly, for Seller 2,

$$\begin{aligned}\max_{Q_2,P_2} \Pi_2(Q_1, P_2|\bar{P}_1) &= P_2(a+\delta\bar{P}_1 - bP_2) \\ &- \frac{A_2(a+\delta\bar{P}_1 - bP_2)}{Q_2} \\ &- C_2(a+\delta\bar{P}_1 - bP_2) \\ &- \frac{1}{2}h_2Q_2 \end{aligned} \tag{16}$$

where $\bar{P}_1$ denotes a given level of per unit price for product 1.

Taking the partial derivative with respect to P_1 and Q_1 of the objective function for (15), we get FONC as

$$\frac{\partial \Pi_1}{\partial P_1} = a - 2bP_1 + \delta\bar{P}_2 + \frac{bA_1}{Q_1} + bC_1 = 0 \tag{17}$$

$$\frac{\partial \Pi_1}{\partial Q_1} = \frac{A_1}{Q_1^2}(a - bP_1 + \delta\bar{P}_2) - \frac{1}{2}H_1 = 0 \tag{18}$$

Meanwhile, the corresponding FONC for (16) are:

$$\frac{\partial \Pi_2}{\partial P_2} = a - 2bP_2 + \delta\bar{P}_1 + \frac{bA_2}{Q_2} + bC_2 = 0 \tag{19}$$

$$\frac{\partial \Pi_2}{\partial Q_2} = \frac{A_2}{Q_2^2}(a - bP_2 + \delta\bar{P}_1) - \frac{1}{2}H_2 = 0 \tag{20}$$

Employing the cubic function formula, we have the following trigonometric forms for

Seller 1.

$$P_1 = \frac{(a + \delta \bar{P}_2)(3 - 2\cos^2 \frac{\theta_1}{3}) + 2bC_1 \cos^2 \frac{\theta_1}{3}}{3b} \tag{21}$$

$$Q_1 = 2[\frac{(a + \delta \bar{P}_2 - bC_1)A_1}{3H_1}]^{\frac{1}{2}} \cos \frac{\theta_1}{3} \tag{22}$$

where

$$\cos \theta_1 = -[\frac{27b^2 H_1 A_1}{4(a + \delta \bar{P}_2 - bC_1)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_1 \leq \frac{3}{4}\pi$.

The corresponding SOSC is given by $4Q_1(a - bP_1 + \delta \bar{P}_2) - bA_1 > 0$.

On the other hand, we obtain the trigonometric forms for Seller 2 as follows.

$$P_2 = \frac{(a + \delta \bar{P}_1)(3 - 2\cos^2 \frac{\theta_2}{3}) + 2bC_2 \cos^2 \frac{\theta_2}{3}}{3b} \tag{23}$$

$$Q_2 = 2[\frac{(a + \delta \bar{P}_1 - bC_2)A_2}{3H_2}]^{\frac{1}{2}} \cos \frac{\theta_2}{3} \tag{24}$$

where

$$\cos \theta_2 = -[\frac{27b^2 H_2 A_2}{4(a + \delta \bar{P}_1 - bC_2)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_2 \leq \frac{3}{4}\pi$.

The corresponding SOSC is given by $4Q_2(a + \delta \bar{P}_1 - bP_2) - bA_2 > 0$.

At an equilibrium point, $P_1 = \bar{P}_1$ and $P_2 = \bar{P}_2$. Hence, the equilibrium point can be

obtained by solving (21), (22), (23), and (24) given $P_1 = \bar{P}_1$ and $P_2 = \bar{P}_2$ for P_1^*, P_2^*, Q_1^*, and Q_2^*. For the basic Bertrand-type model, it has not been possible to obtain a closed form equilibrium point. Therefore, we employ numerical methods to solve for the equilibrium point.

Example 2.

Let A_1 = \$1000, H_1 = \$4.0, C_1 = \$20, A_2 = \$750, H_2 = \$3.0, C_2 = \$15, a=100, b=1.0, and δ=0.5. Then, the corresponding equilibrium point is given by (P_1^*, Q_1^*)=(83.1, 160), (P_2^*, Q_2^*)=(80.4, 175), and Π_1^*=2927, and Π_2^*=3473. Given the same own demand and cross demand effects, the profit difference can be explained by the cost difference.

3. Basic Models under Symmetric Costs

3.1 Cournot-type model with linear demand under symmetric costs

In this section, we assume that the costs of Seller 1 and Seller 2 are symmetric, i.e., $A_1 = A_2$, $C_1 = C_2$, and $H_1 = H_2$. This can be a reasonable assumption for products that differ in color, flavor, etc. With this assumption, we obtain a closed form equilibrium point. And with this closed form equilibrium point, we provide economic implications and managerial insights. Under the assumption of symmetric costs, it can be easily verified that there exists an equilibrium point when $d_1^* = d_2^*$ and $Q_1^* = Q_2^*$. Solving equations (11), (12), (13), and (14) given $d_1 = \bar{d}_1$, $d_2 = \bar{d}_2$, $d_1^* = d_2^*$, and $Q_1^* = Q_2^*$, we obtain the following equation.

$$d^{\frac{3}{2}} - \frac{\alpha - C}{2\beta + \gamma} d^{\frac{1}{2}} + \frac{1}{2\beta + \gamma} (\frac{hA}{2})^{\frac{1}{2}} = 0 \tag{25}$$

From the cubic equation formula, we have the following trigonometric forms for d, Q, and T for Seller 1 and 2.

$$d^* = \frac{4(\alpha - C)}{3(2\beta + \gamma)} \cos^2 \frac{\theta_1}{3} \tag{26}$$

$$Q^* = [\frac{8A(\alpha - C)}{3H(2\beta + \gamma)}]^{\frac{1}{2}} \cos \frac{\theta_1}{3} \tag{27}$$

$$T^* = [\frac{3A(2\beta + \gamma)}{2H(\alpha - C)}]^{\frac{1}{2}} \{\cos \frac{\theta_1}{3}\}^{-1} \tag{28}$$

where

$$\cos \theta_1 = -[\frac{27AH(2\beta + \gamma)}{8(\alpha - C)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_1 \leq \frac{3}{4}\pi$.

SOSC at equilibrium is given by $4Qd\beta - A > 0$.

3.2 Bertrand-type model with linear demand under symmetric costs

Similarly, we assume that the costs of Seller 1 and Seller 2 are symmetric, i.e., $A_1 = A_2$, $C_1 = C_2$, and $H_1 = H_2$. Under the assumption of symmetric costs, it can be easily verified that there exists an equilibrium point when $P_1^* = P_2^*$ and $Q_1^* = Q_2^*$. Solving equations (21), (22), (23), and (24) given $P_1 = \bar{P}_1$, $P_2 = \bar{P}_2$, $P_1^* = P_2^*$, and $Q_1^* = Q_2^*$, we obtain the closed forms as follows:

$$Q^3 - \frac{2A}{h}[\frac{ab-(b-\delta)Cb}{2b-\delta}]Q + \frac{2(b-\delta)bA^2}{h(2b-\delta)} = 0 \tag{29}$$

Applying the cubic equation formula for Trigonometric form, for Seller 1 and 2, we get

$$Q^* = 2[\frac{2A}{3H}\{\frac{ab-(b-\delta)Cb}{2b-\delta}\}]^{\frac{1}{2}} \cos\frac{\theta_2}{3} \tag{30}$$

Substituting the closed form for Q into the FONC, for Seller 1 and 2, we have

$$P^* = \frac{3a(2b-\delta) - 4[ab-(b-\delta)Cb]\cos^2\frac{\theta_2}{3}}{3(b-\delta)(2b-\delta)} \tag{31}$$

$$T^* = \{\frac{3A(2b-\delta)}{2H[ab-(b-\delta)Cb]}\}^{\frac{1}{2}}[\cos\frac{\theta_2}{3}]^{-1} \tag{32}$$

where

$$\cos\theta_2 = -[\frac{27b^2AH(b-\delta)^2(2b-\delta)}{8\{ab-(b-\delta)Cb\}^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_2 \leq \frac{3}{4}\pi$.

SOSC at equilibrium is given by $4Q\{a-(b-\delta)P\} - bA > 0$.

4. Economic Analysis under Symmetric Costs

4.1 Economic analysis for the cournot-type model

Let us examine the sensitivity of decision variables with respect to cost and demand parameters. By differentiating the decision variables at equilibrium with respect to the parameters, we have the following proposition.

Proposition 1: Suppose that the decision point, (d^*, Q^*), under the Cournot-type model, satisfy FONC, SOSC, and equilibrium condition. Then,

$$\frac{\partial Q^*}{\partial A} > 0,\ \frac{\partial Q^*}{\partial H} < 0,\ \frac{\partial Q^*}{\partial C} < 0,\ \frac{\partial Q^*}{\partial \alpha} > 0,\ \frac{\partial Q^*}{\partial \beta} < 0,$$

$$\frac{\partial d^*}{\partial A} < 0,\ \frac{\partial d^*}{\partial H} < 0,\ \frac{\partial d^*}{\partial C} < 0,\ \frac{\partial d^*}{\partial \alpha} > 0,\ \frac{\partial d^*}{\partial \beta} < 0.$$

From the cycle length (in closed form solution) under the Cournot-type model, we have the following proposition.

Proposition 2: Suppose that the decision point, (d^*, Q^*), under the Cournot-type model, satisfy FONC, SOSC, and equilibrium condition. Then, for own demand and cost parameters,

$$\frac{\partial T^*}{\partial A} > 0,\ \frac{\partial T^*}{\partial H} < 0,\ \frac{\partial T^*}{\partial C} > 0\ \frac{\partial T^*}{\partial \alpha} < 0,\ \frac{\partial T^*}{\partial \beta} > 0.$$

We can also obtain the magnitudes of changes in d^* and Q^* with respect to changes

in parameters at equilibrium. For example, it is verified that

$$\frac{\partial d^*}{\partial A} = -\frac{4(\alpha - C)}{9(2\beta + \gamma)} \cos\frac{\theta_1}{3} \times \sin\frac{\theta_1}{3}\left[\frac{\rho}{A(1-\rho^2)^{\frac{1}{2}}}\right] < 0$$

$$\frac{\partial d^*}{\partial H} = -\frac{4(\alpha - C)}{9(2\beta + \gamma)} \cos\frac{\theta_1}{3} \times \sin\frac{\theta_1}{3}\left[\frac{\rho}{H(1-\rho^2)^{\frac{1}{2}}}\right] < 0$$

where

$$\rho = \left[\frac{27AH(2\beta + \gamma)}{8(\alpha - C)}\right]^{\frac{1}{2}}$$

If the the set up cost is much greater than the inventory holding cost per unit per unit time, i.e., H < A, then $\frac{\partial d}{\partial H} < \frac{\partial d}{\partial A} < 0$ at equilibrium. Similarly, numerous analyses can be made on the rest of magnitudes. In this paper, however, we focus on the signs of changes only (due to the page limit; the complete list of signs and magnitudes of changes is available from the authors upon request).

4.2 Economic analysis for the Bertrand-type model

As in the case of the Cournot-type model, by differentiating the decision variables at equilibrium with respect to the parameters, we have the following proposition.

Proposition 3: Suppose that the decision point, (P^*, Q^*), under the Bertrand-type model, satisfy FONC, SOSC, and equilibrium condition. Then,

$$\frac{\partial Q^*}{\partial A} > 0,\ \frac{\partial Q^*}{\partial H} < 0,\ \frac{\partial Q^*}{\partial C} < 0,\ \frac{\partial Q^*}{\partial a} > 0,\ \frac{\partial Q^*}{\partial b} < 0,$$

$$\frac{\partial P^*}{\partial A} > 0,\ \frac{\partial P^*}{\partial H} > 0,\ \frac{\partial P^*}{\partial C} > 0,\ \frac{\partial P^*}{\partial a} > 0,\ \frac{\partial P^*}{\partial b} < 0.$$

To obtain signs of $\frac{\partial P^*}{\partial a}$ and $\frac{\partial P^*}{\partial b}$, we assume that $2(2b-\delta)dQ - Ab(b-\delta) > 0$ and $2Ad + 2cdQ + AbP - 4PdQ > 0$.

From the cycle length (in closed form solution) under the Bertrand-type model, we can summarize the following proposition.

Proposition 4: Suppose that the decision point, (P^*, Q^*), under the Bertrand-type model, satisfy FONC, SOSC, and equilibrium condition. Then, for own demand and cost parameters,

$$\frac{\partial T^*}{\partial A} > 0,\ \frac{\partial T^*}{\partial H} < 0,\ \frac{\partial T^*}{\partial C} > 0\ \frac{\partial T^*}{\partial a} < 0,\ \frac{\partial T^*}{\partial b} > 0.$$

We note that from Proposition 1, 2, 3, and 4, numerous managerial insights can be obtained. For example, $\frac{\partial Q^*}{\partial A} > 0$, $\frac{\partial Q^*}{\partial H} < 0$, and $\frac{\partial Q^*}{\partial C} < 0$ for both the Cournot-type model and the Bertrand-type model. And $\frac{\partial d^*}{\partial A} < 0$, $\frac{\partial d^*}{\partial H} < 0$, and $\frac{\partial d^*}{\partial C} < 0$ for the Cournot-type model. This implies that $\frac{\partial P^*}{\partial A} > 0$, $\frac{\partial P^*}{\partial H} > 0$, and $\frac{\partial P^*}{\partial C} > 0$. This is consistent with $\frac{\partial P^*}{\partial A} > 0$, $\frac{\partial P^*}{\partial H} > 0$, and $\frac{\partial P^*}{\partial C} > 0$, for the Bertrand-type model shown in Proposition 3.

Table 1 Sensitivity analysis I

	A_1	H_1	C_1	α_1	β_1	γ_1
Benchmark	500	4.0	20	100	1.0	0.50
$A_1/A_2 > 1$	750	4.0	20	100	1.0	0.50
$H_1/H_2 > 1$	500	4.5	20	100	1.0	0.50
$C_1/C_2 > 1$	500	4.0	30	100	1.0	0.50
$\alpha_1/\alpha_2 > 1$	500	4.0	20	150	1.0	0.50
$\beta_1/\beta_2 > 1$	500	4.0	20	100	1.5	0.50

5. Numerical Analysis under Symmetric Costs

In this section, we numerically analyze the Cournot-type model and the Bertrand-type model via series of illustrated example.

5.1 Numerical analysis for the Cournot-type model

Example 3.

Let $A_1 = A_2 = \$500$, $H_1 = H_2 = \$4.0$, $C_1 = C_1 = \$20$, α=100, β=1.0, and γ=0.5. Then, the corresponding equilibrium point is given by (d_1^*, Q_1^*)=(29.7, 86.1), (d_2^*, Q_2^*)=(29.7, 86.1), and Π_1^*=709, and Π_2^*=709.

From this benchmark, where $\frac{A_1}{A_2} = 1$, $\frac{H_1}{H_2} = 1$, $\frac{C_1}{C_2} = 1$, $\frac{\alpha_1}{\alpha_2} = 1$, $\frac{\beta_1}{\beta_2} = 1$, and $\frac{\gamma_1}{\gamma_2} = 1$, we vary the parameter values of Seller 1 while we keep the parameter values of Seller 2 as the same. Table 1 summarizes such changes. The resulting equilibrium points due to these changes are summarized in Table 2.

From the above two tables, numerous observations can be made for managerial insights. For example, if the set up cost for own product is increased from 500 to 750, then the sales quantity per unit time will decrease while the order quantity per cycle

Table 2 Sensitivity analysis II

	d_1^*	d_2^*	Q_1^*	Q_2^*	Π_1^*	Π_2^*
Benchmark	29.7	29.7	86.1	86.1	709	709
$A_1/A_2 > 1$	28.9	29.9	104.2	86.4	629	720
$H_1/H_2 > 1$	29.5	29.7	80.9	86.2	687	711
$C_1/C_2 > 1$	24.0	31.2	77.4	88.3	420	795
$\alpha_1/\alpha_2 > 1$	57.3	22.3	120.0	74.7	3047	349
$\beta_1/\beta_2 > 1$	18.8	32.5	68.6	90.1	394	877

Table 3 Sensitivity analysis III

	A_1	H_1	C_1	a_1	b_1	δ_1
Benchmark	500	4.0	20	100	1.0	0.50
$A_1/A_2 > 1$	750	4.0	20	100	1.0	0.50
$H_1/H_2 > 1$	500	4.5	20	100	1.0	0.50
$C_1/C_2 > 1$	500	4.0	30	100	1.0	0.50
$a_1/a_2 > 1$	500	4.0	20	150	1.0	0.50
$b_1/b_2 > 1$	500	4.0	20	100	1.5	0.50

will increase. Hence, the change of the cycle length is determined as being longer.

5.2 Numerical analysis for the Bertrand-type model

Next, the following example for the Bertrand-type model is illustrated.

Example 4.

Let $A_1 = A_2 = \$500$, $H_1 = H_2 = \$4.0$, $C_1 = C_1 = \$20$, a=100, b=1.0, and δ=0.5. Then, the corresponding equilibrium point is given by (P_1^*, Q_1^*)=(82.8, 121.1), (P_2^*, Q_2^*)=(82.8, 121.1), and Π_1^*=3195, and Π_2^*=3195. From this benchmark, where $\frac{A_1}{A_2} = 1$, $\frac{H_1}{H_2} = 1$, $\frac{C_1}{C_2} = 1$, $\frac{a_1}{a_2} = 1$, $\frac{b_1}{b_2} = 1$, and $\frac{\delta_1}{\delta_2} = 1$, we vary the parameter values of Seller 1 while we keep the parameter values of Seller 2 as the same. Table 3 summarizes such changes. The resulting equilibrium points due to these changes are summarized in Table 4.

We note that the signs of changes in decision variables in this section are consistent

Table 4 Sensitivity analysis IV

	P_1^*	P_2^*	Q_1^*	Q_2^*	Π_1^*	Π_2^*
Benchmark	82.8	82.8	121.1	121.1	3195	3195
$A_1/A_2 > 1$	83.3	82.9	147.7	121.2	3090	3209
$H_1/H_2 > 1$	82.9	82.8	114.0	121.1	3166	3199
$C_1/C_2 > 1$	88.2	84.1	116.0	122.5	2670	3355
$a_1/a_2 > 1$	109.0	89.2	146.3	127.8	7034	4008
$b_1/b_2 > 1$	58.3	76.8	112.8	114.5	1499	2517

with the results from Proposition 1, 2, 3, and 4 of the previous section.

6. Concluding Remarks

We designed and analyzed two duopoly models for substitute products. For both the Cournot-type model and the Bertrand-type model, we showed how the optimal inventory and pricing policies were derived from the first order necessary conditions. We further showed how the inventory and pricing policies were obtained at equilibrium. Next, under the assumption of symmetric costs, we obtained the closed form inventory and pricing policies at equilibrium. From the closed form policies at equilibrium, numerous economic implications were obtained via calculus and numerical analyses. The basic models in this paper can be extended by considering such features as three or more sellers, three or more products, and nonlinear demand and/or inverse demand functions.

References

Beyer, W. H. (1981). *Standard Mathematical Table.* Boca Raton, Florida: CRC Press.

Chen, Cheng-Kang, and Min, K. Jo (1994). An analysis of optimal inventory and pricing policies under linear demand. *Asia-Pacific Journal of Operational Research, 11*, 117–129.

Chen, Cheng-Kang, and Min, K. Jo (1995). A Competitive Inventory Model with Options to Reduce Setup and Inventory Holding Costs. *Computers and Operations Research, 11*, 503–514.

Choi, Chan (1991). Price Competition in A Channel Structure with A Common Retailer. *Marketing Science, 10*, 271–295.

Min, K. Jo. (1992). Inventory and pricing policies under competition. *Operations Research Letters, 12*, 253–261.

Varian, H. R. (1992). *Microeconomic Analysis*, New York, New York: W. W. Norton & Company.

CHAPTER 6. INVENTORY AND PRICING POLICIES FOR A DUOPOLY OF COMPLEMENTS

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Toshitsugu Otake and K. Jo Min

Abstract

We design and analyze two duopoly models for two competing sellers. Each seller is assumed to be a profit maximizing EOQ-based decision maker facing linear demand functions. We also assume that a single product is sold by each seller, and the two products of the two sellers are complements. Under these assumptions, in the first duopoly model, we develop a Cournot-type duopoly model where competition is over the selling quantity. In the second duopoly model, we develop a Bertrand-type duopoly model where competition is over the selling price. For both models, we derive and analyze equilibrium inventory and pricing policies. Various interesting numerical examples are illustrated.

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Keywords

Economic Order Quantity, Duopoly, Complements, and Pricing.

1. Introduction

In the inventory literature, we find numerous papers examining the economic implications of pricing and inventory policies under competition. For example, Min (1992) investigates both uniform and quantity discount pricing and inventory policies under competition. Recently, Otake and Min (1995) extend Min (1992) by considering two substitute products where an increase in one product's price results in an increase in another product's demand. In this paper, we examine a parallel case of Otake and Min (1995). Namely, an increase in one product's price results in a decrease in another products demand. i.e., the two products are complement (e.g., tennis rackets and tennis balls). Even though there have been extensive studies of complements in the literature of economic theory, to our knowledge, there have been few papers dealing with complements in the context of inventory policies (competitive or otherwise). Hence, given the prevalence of complements in the real world, it is highly desirable to derive economic implications and managerial insights in the context of inventory.

In this paper, based on Cournot-type and Bertrand-type competitive behavioral assumptions (see e.g., Mas-Collel et al. 1995; Varian 1992), we design and analyze pricing and inventory policies for two sellers. Each seller is assumed to produce a single product and maximize his profit and the product of one seller is a complement to the product of the other seller.

Specifically, under the Cournot-type model, each seller chooses his demand (i.e., sale) per unit time as a decision variable in order to maximize his profit per unit time given his expectation on the level of demand (i.e., sale) per unit time of the other seller. On the other hand, under the Bertrand-type model, each seller chooses his price per unit as a decision variable in order to maximize his profit per unit time given his expectation on the level of price per unit of the other seller. Because of dependency of demand and price, the pricing policies determine the sale policies: conversely, the sale policies determine the price policies. Dependency of demand and price are expressed by the linear demand functions, which are widely found in the literature of economics (see Choi 1991; Vives 1985).

Under these assumptions, we first derive the equilibrium conditions for both Cournot-type and Bertrand-type Models. Next, assuming the symmetric demand and cost functions, we derive the closed form solutions and analyze inventory and pricing policies in depth. Finally, we derive various interesting managerial insights and economic implications. For example, the Bertrand-type competition results in higher total sale per unit time than the Cournot-type competition, which is consistent with the outcome shown in the literature of economics.

2. Two Types of Basic Models

2.1 Definitions and assumptions

For the Cournot-type model, the following linear inverse demand function is utilized.

$$p_1 = \alpha_1 - \beta_1 d_1 + \gamma d_2 \tag{1}$$

$$p_2 = \alpha_2 + \gamma d_1 - \beta_2 d_2 \tag{2}$$

where

p_i: the per unit price of product i,

d_i: the per unit time demand of product i,

α_i: the intercept of the inverse demand function

β_i: the own price effect

γ: the cross price effect $\forall$ i=1 and 2,

and parameters, α_i, β_i, and γ are positive. The cross price effects are symmetric as is required for well-behaved consumer demand function (see Varian 1992). Furthermore, $\frac{\gamma^2}{\beta_1\beta_2}$ represents the degree of product differentiation and which is between 0 and 1. Hence, γ^2 must be less than or equal to the product of β_1 and β_2 (see Choi 1991).

Similarly, for the Bertrand-type model, the following direct linear demand function is employed.

$$d_1 = a_1 - b_1 p_1 - \delta p_1 \tag{3}$$

$$d_2 = a_2 - \delta p_1 - b_2 p_2 \tag{4}$$

where

a_i: the intercept of the demand function

b_i: the own demand effect

δ: the cross demand effect $\forall$ i=1 and 2,

and parameters, a_i, b_i, and δ, are positive and the cross demand effects are symmetric. Also, the sum of the product of the intercept for product i and the own price effect for product j and the product of the intercept for the product j and the cross price effect is greater than zero for $i, j = 1, 2$ *and* $i \neq j$ (see Vives 1985). The relations among the parameters of the demand functions and inverse demand functions are as follows:

$$a_i = \frac{\alpha_i \beta_j + \alpha_j \gamma}{\beta_i \beta_j - \gamma^2} \tag{5}$$

$$b_i = \frac{\beta_j}{\beta_i \beta_j - \gamma^2} \tag{6}$$

$$\delta = \frac{\gamma}{\beta_i \beta_j - \gamma^2} \tag{7}$$

where i, j=1, 2 and $i \neq j$.

In order to mathematically formulate these models, the following variables and pa-

rameters are defined: For i = 1, 2,

Q_i: the order quantity of product i for Seller i

A_i: the set up cost of product i

c_i: the variable cost per unit of product i

h_i: the inventory holding cost per unit per unit

time of product i

T_i: the cycle length for product i.

Also, by the definition of complements (see e.g., Varian 1992), we assume $\frac{\partial d_i}{\partial p_i} < 0$ and $\frac{\partial d_i}{\partial p_j} < 0$ for i, j=1, 2.

The basic assumptions for traditional EOQ model applied in this paper are as follows.

1) buyer's demand rate is constant over time,

2) the replenishment rate is infinite,

3) no shortage is allowed,

4) there is no delivery lag.

2.2 Cournot-type model

Under the definitions and assumptions shown above, we design and analyze the Cournot-type model with the linear demand function as follows. For Seller i, i=1 and 2, the per unit time profit maximization problem is:

$$\max_{Q_i,d_i} \Pi_i(Q_i, d_i|\bar{d}_j) = (\alpha_i - \beta_i d_i + \gamma \bar{d}_j)d_i - \frac{A_i d_i}{Q_i} - c_i d_i - \frac{1}{2}h_i Q_i \quad (8)$$

where $\bar{d}_j$ denotes a given level of demand per unit time for Seller j and i, j=1, 2 and $i \neq j$.

The corresponding first order necessary conditions (FONC) for (8) are:

$$\frac{\partial \Pi_i}{\partial d_i} = \alpha_i - 2\beta_i d_i + \gamma \bar{d}_j - \frac{A_i}{Q_i} - c_i = 0 \quad (9)$$

$$\frac{\partial \Pi_i}{\partial Q_i} = \frac{A_i d_i}{Q_i^2} - \frac{1}{2}h_i = 0 \quad (10)$$

From the cubic equation formula in the Standard Mathematical Table (Beyer 1981), we have the following solutions for d_i and Q_i, i=1,2, given $\bar{d}_j$, $j \neq i$.

$$d_i = \frac{2(\alpha_i + \gamma \bar{d}_j - c_i)}{3\beta_i} \cos^2 \frac{\theta_i}{3} \quad (11)$$

$$Q_i = \left[\frac{4A_i(\alpha_i + \gamma \bar{d}_j - c_i)}{3\beta_i h_i}\right]^{\frac{1}{2}} \cos \frac{\theta_i}{3} \quad (12)$$

where

$$\cos \theta_i = -\left[\frac{27\beta_i h_i A_i}{4(\alpha_i + \gamma \bar{d}_j - c_i)^3}\right]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_i \leq \frac{3}{4}\pi$ assuming non-negative profit (see, e.g., Chen and Min 1994).

The corresponding second order sufficient condition (SOSC) is given by

$$4Q_i d_i \beta_i - A_i > 0.$$

We note that (11) and (12) are the reaction functions illustrating the optimal choices for Seller i given Seller j's decision on $\bar{d}_j$.

2.3 Bertrand-type model

Likewise, for Seller i, i=1,2, the per unit time profit maximization problem is as follows.

$$\begin{aligned} \max_{Q_i, p_i} \Pi_i(Q_i, p_i | \bar{p}_j) &= p_i(a_i - b_i p_i - \delta \bar{p}_j) \\ &- \frac{A_i(a_i - b_i p_i - \delta \bar{p}_j)}{Q_i} \\ &- c_i(a_i - b_i p_i - \delta \bar{p}_j) \\ &- \frac{1}{2} h_i Q_i \end{aligned} \tag{13}$$

where $\bar{p}_j$ denotes a given level of per unit price for product j and i, j=1, 2 and $i \neq j$.

Taking the partial derivative with respect to p_i and Q_i of the objective function for (13), we get FONC as

$$\frac{\partial \Pi_i}{\partial p_i} = a_i - 2b_i p_i - \delta \bar{p}_j + \frac{b_i A_i}{Q_i} + b_i c_i = 0 \tag{14}$$

$$\frac{\partial \Pi_i}{\partial Q_i} = \frac{A_i}{Q_i^2}(a_i - b_i p_i - \delta \bar{p}_j) - \frac{1}{2} h_i = 0 \tag{15}$$

Employing the cubic function formula, we have the following trigonometric forms for Seller 1.

$$p_i = \frac{(a_i - \delta \bar{p}_j)(3 - 2\cos^2 \frac{\theta_i}{3}) + 2b_i c_i \cos^2 \frac{\theta_i}{3}}{3b_i} \tag{16}$$

$$Q_i = 2[\frac{(a_i - \delta \bar{p}_j - b_i c_i) A_i}{3h_i}]^{\frac{1}{2}} \cos \frac{\theta_i}{3} \tag{17}$$

where

$$\cos \theta_i = -[\frac{27 b_i^2 h_i A_i}{4(a_i - \delta \bar{p}_j - b_i c_i)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_i \leq \frac{3}{4}\pi$.

The corresponding SOSC is given by

$$4Q_i(a_i - b_i p_i - \delta \bar{p}_j) - b_i A_i > 0.$$

We note that (16) and (17) are the reaction functions illustrating the optimal choices

for Seller i given Seller j's decision on $\bar{p}_j$. We also note that given reaction functions of (11) and (12) or (16) and (17), it is possible to obtain equilibrium points numerically. For example, we can solve numerically (11) and (12), i=1,2, where $d_1 = \bar{d}_1$ and $d_2 = \bar{d}_2$ for d_1^*, d_2^*, Q_1^* and Q_2^* for a Cournot-type equilibrium solution. Likewise, we can solve numerically (16) and (17), i=1,2, where $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$ for p_1^*, p_2^*, Q_1^* and Q_2^* for a Bertrand-type equilibrium solution. In the next subsection, we proceed to assume symmetric demand and cost functions, and solve for an equilibrium solution in closed form.

3. Basic Models under Symmetric Data

3.1 Cournot-type model under symmetric cost and inverse demand functions

In this section, we assume that the inverse demand and the cost functions of Seller 1 and Seller 2 are symmetric, i.e., $A_1 = A_2 = A$, $c_1 = c_2 = c$, $h_1 = h_2 = h$, $\alpha_1 = \alpha_2 = \alpha$, and $\beta_1 = \beta_2 = \beta$. Under the assumption of symmetric cost and inverse demand functions, it can be easily verified that there exists an equilibrium point where $d_1^* = d_2^*$ and $Q_1^* = Q_2^*$. Solving equations (11) and (12) given $d_1 = \bar{d}_1$, $d_2 = \bar{d}_2$, $d_1^* = d_2^*$, and $Q_1^* = Q_2^*$, we obtain the following equation.

$$d_c^* = \frac{4(\alpha - c)}{3(2\beta - \gamma)} \cos^2 \frac{\theta_1}{3} \tag{18}$$

$$Q_c^* = [\frac{8A(\alpha - c)}{3h(2\beta - \gamma)}]^{\frac{1}{2}} \cos \frac{\theta_1}{3} \tag{19}$$

$$T_c^* = [\frac{3A(2\beta - \gamma)}{2h(\alpha - c)}]^{\frac{1}{2}} \{\cos \frac{\theta_1}{3}\}^{-1} \tag{20}$$

where

$$\cos \theta_1 = -[\frac{27Ah(2\beta - \gamma)}{8(\alpha - c)^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_1 \leq \frac{3}{4}\pi$.

SOSC at equilibrium is given by

$$4Q_c^* d_c^* \beta - A > 0.$$

3.2 Bertrand-type model under symmetric cost and demand function

Similarly, we assume that the cost and demand functions of Seller 1 and Seller 2 are symmetric, i.e., $A_1 = A_2 = A$, $c_1 = c_2 = c$, $h_1 = h_2 = h$, $a_1 = a_2 = a$, $b_1 = b_2 = b$, and $\delta_1 = \delta_2 = \delta$. Under the assumption of symmetric cost and demand functions, it can be easily verified that there exists an equilibrium solutions where $p_1^* = p_2^*$ and $Q_1^* = Q_2^*$. Solving equations (16) and (17) given $p_1 = \bar{p}_1$, $p_2 = \bar{p}_2$, $p_1^* = p_2^*$, and $Q_1^* = Q_2^*$, we obtain the closed forms as follows.

$$Q_b^* = [\frac{8A}{3h}\{\frac{ab-(b+\delta)cb}{2b+\delta}\}]^{\frac{1}{2}} \cos\frac{\theta_2}{3} \tag{21}$$

Substituting the closed form for Q into the FONC, for Seller 1 and 2, we have

$$p_b^* = \frac{3a(2b+\delta)-4[ab-(b+\delta)cb]\cos^2\frac{\theta_2}{3}}{3(b+\delta)(2b+\delta)} \tag{22}$$

$$T_b^* = \{\frac{3A(2b+\delta)}{2h[ab-(b+\delta)cb]}\}^{\frac{1}{2}}[\cos\frac{\theta_2}{3}]^{-1} \tag{23}$$

where

$$\cos\theta_2 = -[\frac{27b^2Ah(b+\delta)^2(2b+\delta)}{8\{ab-(b+\delta)cb\}^3}]^{\frac{1}{2}}$$

and $\frac{1}{2}\pi < \theta_2 \leq \frac{3}{4}\pi$.

SOSC at equilibrium is given by

$$4Q_b^*\{a-(b+\delta)p_b^*\} - bA > 0.$$

4. Economic Analysis

4.1 Economic analysis for the Cournot-type model

Let us examine the sensitivity of decision variables with respect to cost and inverse demand parameters numerically and analytically. By differentiating the decision variables at equilibrium with respect to the parameters, we have the following proposition.

Proposition 1: Suppose that the decision point, (d^*, Q^*), under the Cournot-type model, satisfies FONC, SOSC, and equilibrium condition. Then,

$$\frac{\partial Q_c^*}{\partial A} > 0,\ \frac{\partial Q_c^*}{\partial h} < 0,\ \frac{\partial Q_c^*}{\partial c} < 0,\ \frac{\partial Q_c^*}{\partial \alpha} > 0,\ \frac{\partial Q_c^*}{\partial \beta} < 0,$$
$$\frac{\partial d_c^*}{\partial A} < 0,\ \frac{\partial d_c^*}{\partial h} < 0,\ \frac{\partial d_c^*}{\partial c} < 0,\ \frac{\partial d_c^*}{\partial \alpha} > 0,\ \frac{\partial d_c^*}{\partial \beta} < 0.$$

From the cycle length (in closed form solution) under the Cournot-type model, we have the following proposition.

Proposition 2: Suppose that the decision point, (d^*, Q^*), under the Cournot-type model, satisfies FONC, SOSC, and equilibrium condition. Then, for own demand and cost parameters,

$$\frac{\partial T_c^*}{\partial A} > 0,\ \frac{\partial T_c^*}{\partial h} < 0,\ \frac{\partial T_c^*}{\partial c} > 0\ \frac{\partial T_c^*}{\partial \alpha} < 0,\ \frac{\partial T_c^*}{\partial \beta} > 0.$$

We can also obtain the magnitudes of changes in d^* and Q^* with respect to changes in various parameters at equilibrium. For example, it is verified that

$$\frac{\partial d_c^*}{\partial \alpha} = \frac{4}{3(2\beta-\gamma)}\cos^2\frac{\theta_1}{3} + \frac{4\rho}{3(2\beta-\gamma)\sqrt{1-\rho^2}}\cos\frac{\theta_1}{3}\sin\frac{\theta_1}{3} > 0 \quad (24)$$

$$\frac{\partial d_c^*}{\partial \beta} = -\frac{8(\alpha-c)}{3(2\beta-\gamma)^2}\cos^2\frac{\theta_1}{3} - \frac{8(\alpha-c)\rho}{9(2\beta-\gamma)^2\sqrt{1-\rho^2}}\cos\frac{\theta_1}{3}\sin\frac{\theta_1}{3} < 0 \quad (25)$$

where

$$\rho = [\frac{27Ah(2\beta-\gamma)}{8(\alpha-c)^3}]^{\frac{1}{2}}$$

(24) represents the change of magnitude in d_i^* when the intercept of inverse demand increases infinitesimally. Also, (25) represents the change magnitude in d_i^* when the own price effect of inverse demand increases infinitesimally.

4.2 Economic analysis for the Bertrand-type model

Likewise, we have the following propositions.

Proposition 3: Suppose that the decision point, (p^*, Q^*), under the Bertrand-type model, satisfies FONC, SOSC, and equilibrium condition. Then,

$$\frac{\partial Q_b^*}{\partial A} > 0,\ \frac{\partial Q_b^*}{\partial h} < 0,\ \frac{\partial Q_b^*}{\partial c} < 0,\ \frac{\partial Q_b^*}{\partial \alpha} > 0,\ \frac{\partial Q_b^*}{\partial \beta} < 0.$$
$$\frac{\partial P_b^*}{\partial A} > 0,\ \frac{\partial P_b^*}{\partial h} > 0,\ \frac{\partial P_b^*}{\partial c} > 0,\ \frac{\partial P_b^*}{\partial a} > 0,\ \frac{\partial P_b^*}{\partial b} < 0.$$

To obtain signs of $\frac{\partial P^*}{\partial a}$ and $\frac{\partial P^*}{\partial b}$, we assume that $2(2b+\delta)dQ - Ab(b+\delta) > 0$ and $2Ad + 2cdQ + AbP - 4PdQ > 0$.

From the cycle length (in closed form solution) under the Bertrand-type model, we can summarize the following proposition.

Proposition 4: Suppose that the decision point, (p^*, Q^*), under the Bertrand-type model, satisfies FONC, SOSC, and equilibrium condition. Then, for own demand and cost parameters,

$$\frac{\partial T_b^*}{\partial A} > 0,\ \frac{\partial T_b^*}{\partial h} < 0,\ \frac{\partial T_b^*}{\partial c} > 0\ \frac{\partial T_b^*}{\partial a} < 0,\ \frac{\partial T_b^*}{\partial b} > 0.$$

We note that from Proposition 1, 2, 3, and 4, numerous managerial insights can be obtained. For example, $\frac{\partial Q^*}{\partial A} > 0$, $\frac{\partial Q^*}{\partial h} < 0$, and $\frac{\partial Q^*}{\partial c} < 0$ for both the Cournot-type model and the Bertrand-type model. And $\frac{\partial d^*}{\partial A} < 0$, $\frac{\partial d^*}{\partial h} < 0$, and $\frac{\partial d^*}{\partial c} < 0$ for the Cournot-type model. This implies that $\frac{\partial P^*}{\partial A} > 0$, $\frac{\partial P^*}{\partial h} > 0$, and $\frac{\partial P^*}{\partial c} > 0$. This is consistent with $\frac{\partial P^*}{\partial A} > 0$, $\frac{\partial P^*}{\partial h} > 0$, and $\frac{\partial P^*}{\partial c} > 0$, for the Bertrand-type model shown in Proposition 3. Economic interpretations are straightforward. e.g., the increase in the setup cost leads to the increase in the order quantity, the decrease in demand, and the increase in price.

Table 1 Sensitivity analysis I

	A_1	h_1	c_1	α_1	β_1	γ_1
Benchmark	500	4.0	20	100	1.0	0.50
$A_1/A_2 > 1$	505	4.0	20	100	1.0	0.50
$h_1/h_2 > 1$	500	4.04	20	100	1.0	0.50
$c_1/c_2 > 1$	500	4.0	20.2	100	1.0	0.50
$\alpha_1/\alpha_2 > 1$	500	4.0	20	101	1.0	0.50
$\beta_1/\beta_2 > 1$	500	4.0	20	100	1.01	0.50

5. Numerical Analysis under Symmetric Cost and Demand/Inverse Demand Functions

In this section, we numerically analyze the Cournot-type model and the Bertrand-type model via a series of illustrated examples.

5.1 Numerical analysis for the Cournot-type model

Example 1.

Let $A_1 = A_2 = \$500$, $h_1 = h_2 = \$4.0$, $c_1 = c_1 = \$20$, α=100, β=1.0, and γ=0.5. Then, the corresponding equilibrium point is given by (d_1^*, Q_1^*)=(50.4, 112.2), (d_2^*, Q_2^*)=(50.4, 112.2), and Π_1^*=2312, and Π_2^*=2312.

From this benchmark, where $\frac{A_1}{A_2} = 1$, $\frac{h_1}{h_2} = 1$, $\frac{c_1}{c_2} = 1$, $\frac{\alpha_1}{\alpha_2} = 1$, $\frac{\beta_1}{\beta_2} = 1$, and $\frac{\gamma_1}{\gamma_2} = 1$, we vary the parameter values of Seller 1 by 1 percent of the given values while we keep the parameter values of Seller 2 as the same. Table 1 summarizes such changes.

The percentage change of the resulting equilibrium points due to above changes from the Benchmark value are summarized in Table 2.

From the above two tables, numerous observations can be made for managerial in-

Table 2 Sensitivity analysis II

	d_1^*	d_2^*	Q_1^*	Q_2^*	Π_1^*	Π_2^*
Benchmark	50.4	50.4	112.2	112.2	2312	2312
$A_1/A_2 > 1$	-0.02	-0.01	0.49	-0.003	-0.1	-0.01
$h_1/h_2 > 1$	-0.02	-0.01	-0.51	-0.003	-0.4	-0.01
$c_1/c_2 > 1$	-0.22	-0.06	-0.11	-0.03	-0.13	-0.01
$\alpha_1/\alpha_2 > 1$	1.09	0.28	0.54	0.14	2.3	0.6
$\beta_1/\beta_2 > 1$	-1.1	-0.28	-0.54	-0.13	-1.23	-0.59

sights. For example, if the set up cost for our own product is increased from 500 to 505, then the sales quantity per unit time will decrease while the order quantity per cycle will increase.

5.2 Numerical analysis for the Bertrand-type model

Next, the following example for the Bertrand-type model is illustrated.

Example 2.

Let $A_1 = A_2 = \$500$, $h_1 = h_2 = \$4.0$, $c_1 = c_1 = \$20$, and $\alpha = 100$, $\beta = 1$, and $\gamma = 0.5$, that is, a=200, b=$\frac{4}{3}$, and $\delta = \frac{2}{3}$ by (5) through (7). Then, the corresponding equilibrium point is given by (p_1^*, Q_1^*)=(69.6, 123.2), (p_2^*, Q_2^*)=(69.6, 123.2), and Π_1^*=2522, and Π_2^*=2522. From this benchmark, where $\frac{A_1}{A_2} = 1$, $\frac{h_1}{h_2} = 1$, $\frac{c_1}{c_2} = 1$, $\frac{a_1}{a_2} = 1$, $\frac{b_1}{b_2} = 1$, and $\frac{\delta_1}{\delta_2} = 1$, we vary the parameter values of Seller 1 while we keep the parameter values of Seller 2 as the same. Table 3 summarizes such changes. The resulting equilibrium points due to above changes are summarized in Table 4 .

We note that the signs of changes in decision variables in this section are consistent with the results from Proposition 1, 2, 3, and 4 of the previous section. From these examples, with the equivalent parameter values, we find that the total profit for the

Table 3 Sensitivity analysis III

	A_1	h_1	c_1	a_1	b_1	δ_1
Benchmark	500	4.0	20	200	1.333	0.666
$A_1/A_2 > 1$	505	4.0	20	200	1.333	0.666
$h_1/h_2 > 1$	500	4.04	20	200	1.333	0.666
$c_1/c_2 > 1$	500	4.0	20.2	200	1.333	0.666
$\alpha_1/\alpha_2 > 1$	500	4.0	20	201.333	1.333	0.666
$\beta_1/\beta_2 > 1$	500	4.0	20	197.368	1.3158	0.658

Table 4 Sensitivity analysis IV

	p_1^*	p_2^*	Q_1^*	Q_2^*	Π_1^*	Π_2^*
Benchmark	69.6	69.6	123.2	123.2	2522	2522
$A_1/A_2 > 1$	0.02	-0.004	0.49	-0.003	-0.1	-0.01
$h_1/h_2 > 1$	0.02	-0.004	-0.51	-0.003	-0.4	-0.01
$c_1/c_2 > 1$	0.16	-0.04	-0.11	-0.03	-0.1	-0.13
$\alpha_1/\alpha_2 > 1$	0.66	0.19	0.52	0.15	-0.2	-0.55
$\beta_1/\beta_2 > 1$	0.05	-0.11	-0.65	-0.26	0.1	-0.04

Bertrand is higher than that for the Cournot. Hence, in a real-life setting, each firm has an incentive to induce price-competition rather than quantity-competition. Furthermore, when we observe the demand level under the Bertrand-type model and under the Cournot-type model, we can obtain the following relations.

$$d_{cs}^* < d_{bs}^* \tag{26}$$

$$d_{cc}^* < d_{bc}^* \tag{27}$$

$$p_{bs}^* < p_{cs}^* \tag{28}$$

$$p_{bc}^* < p_{cc}^* \tag{29}$$

where the first letter c or b of the subscript stands for the Cournot-type Model or the Bertrand-type Model, respectively. Likewise, the second letter s or c represents substitutes or complements, respectively. In the theory of economics, this implies that the Cournot-type model is more monopolistic competition than the Bertrand-type model (see Vives 1985).

6. Comparison and Contrast of Substitutes with Complements

We are currently in the process of comparing and contrasting the outcomes from substitute products (see Otake and Min 1995) and from complements. In particular, under symmetric cost and demand/inverse demand functions, we define the following:

S_{cc}: a critical quantity of complements under Cournot-type model,

S_{bc}: a critical quantity of complements under Bertrand-type model,

S_{cs}: a critical quantity of substitutes under Cournot-type model,

S_{bs}: a critical quantity of substitutes under Bertrand-type model.

A critical quantity here can represent p, d, Q, and T.

By mathematical manipulation of the order quantities and the cycle length for both substitute and complement cases, we can claim

$$\frac{Q^*_{cs}}{Q^*_{cc}} < 1, \qquad \frac{Q^*_{bs}}{Q^*_{bc}} < 1 \tag{30}$$

$$\frac{T^*_{cc}}{T^*_{cs}} < 1, \qquad \frac{T^*_{bc}}{T^*_{bs}} < 1 \tag{31}$$

Condition (30) states that the EOQ for complements under both the Cournot-type and Bertrand-type Models are greater than the EOQ for substitutes. Hence, the amplitude of cycle length for complements, that is the EOQ level for complements, is higher than that for substitutes. On the other hand, condition (31) states that the cycle length for substitutes are longer than that for complements.

7. Concluding Remarks

In this paper, we developed and analyzed two duopoly models for complements. From the first order necessary and the second order sufficient conditions, it was shown how the optimal inventory and pricing policies were derived. As a special case, the symmetric demand and cost were assumed and the closed form inventory and pricing policies were obtained at equilibrium. Comparing substitutes with complements, we showed that the Bertrand-type competition was more efficient than the Cournot-type competition and the sellers tended to have higher EOQ for complements than that for substitutes. This paper can be extended by designing and analyzing different market behavioral assumptions, such as the Stackelberg model and the price leadership with several sellers or several products (see e.g.. Mas-Collel 1995; Varian 1992). Furthermore, nonlinear demand functions and inverse demand functions need to be addressed.

References

Beyer, W. H. (1981). *Standard Mathematical Table.* Boca Raton, Florida: CRC Press.

Chen, Cheng-Kang, and Min, K. Jo (1994). An analysis of optimal inventory and pricing policies under linear demand. *Asia-Pacific Journal of Operational Research, 11*, 117–129.

Choi, Chan. (1991). Price Competition in A Channel Structure with A Common Retailer. *Marketing Science, 10*, 271–295.

Mas-Collel, Andreu, Whinston, Michael D., and Green, Jerry R. (1995). *Microeconomic Theory*, New York, New York: Oxford University Press, Inc.

Min, K. Jo. (1992). Inventory and pricing policies under competition. *Operations Research Letters, 12*, 253–261.

Otake, Toshitsugu, and Min, K. Jo (1995). Inventory and Pricing Policies for a Duopoly of Substitute Products. *The Proceedings of 5th IERC, 11*, 293–298.

Varian, H. R. (1992). *Microeconomic Analysis*, New York, New York: W. W. Norton & Company.

Vives, Xavier. (1985). On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation. *Journal of Economic Theory, 36*, 166–175.

GENERAL CONCLUDING REMARKS

Summary of Dissertation

In this dissertation, inventory, investment, and pricing policies for lot-size decision makers were examined based on classical economic order quantity. Specifically, we focused on investment in setup operations, investment in quality improvement, and market dependent products such as substitutes and complements. We examined various impacts of investment and competition on inventory policies and derived managerial insights and economic implications. Throughout this dissertation, deterministic mathematical programming was used as the primary analysis technique and optimal policies were obtained through this technique.

In order to investigate the impact of investment, first, we focused on inventory and investment in quality improvement under ROI maximization. Next, we focused on inventory and investment in setup operations under return on investment (ROI) maximization. Also, we were investigating inventory and capital investment allocation policies in setup and quality operations under ROI maximization. Furthermore, we were comparing and contrasting inventory and investment policies under ROI maximization with those

policies under other economic/finance performance criteria such as profit maximization.

In order to investigate the impact of competition, on the other hand, we first designed and analyzed two duopoly models for two profit maximizing sellers when products are substitute. we also designed and analyzed two duopoly models for two profit maximizing sellers when products are complement. Furthermore, we compared and contrasted these models.

In each chapter, there were several interesting managerial insights and economic implications and numerical examples were illustrated. We conclude this dissertation by summarizing contents in each chapter below.

In Chapter 1, we constructed and analyzed inventory and investment in quality improvement policies under ROI maximization. Specifically, first, we showed how an ROI maximization problem is formulated. Next, the unique global optimal solution is determined by employing the primary criterion of ROI maximization and the secondary criterion of the prior order quantity minimization. In addition, we showed how the levels for the prior and posterior order quantities are reduced when it is optimal to invest additional money in quality improvement.

In Chapter 2, we constructed and analyzed inventory and investment in setup operations policies under return on investment (ROI) maximization. Specifically, we showed how an ROI maximization problem is formulated and the unique global optimal solution is determined. Furthermore, we showed how the inventory level is reduced when it is optimal to invest additional money in setup operations. There are several extensions that will further enhance the importance and relevance of our model.

In Chapter 3, we constructed and analyzed inventory and capital investment allocation policies under return on investment (ROI) maximization. Our model was constructed for a decision maker of a single product with a budget constraint in capital investment. We showed how the levels for the prior and posterior order quantities are reduced when it is optimal to invest additional money in setup cost reduction and/or quality improvement. An illustrated numerical example was provided in order to show sensitivity analysis of unit variable cost.

In Chapter 4, we investigated inventory policies and investment in setup operations policies under profit maximization and ROI maximization. First, we studied how a profit maximization problem and an ROI maximization problem are formulated. Second, we examined the unique global optimal solution by the primary criterion of profit maximization or ROI maximization and the secondary criterion of the order quantity minimization. Furthermore, by employing the secondary criterion, we compared and contrasted the unique global optimal solutions under profit and ROI maximization. Finally, under the assumption of rational and linear setup cost functions, we first obtained the unique global optimal solutions and provided the decision making rules to determine the unique global optimal solution.

In Chapter 5, we designed and analyzed two duopoly models for substitute products. We showed how the optimal inventory and pricing policies were derived from the first order necessary conditions for both the Cournot-type model and the Bertrand-type model. We further showed how the inventory and pricing policies were obtained at equilibrium. Next, under the assumption of symmetric costs, we obtained the closed form inventory

and pricing policies at equilibrium.

In Chapter 6, we developed and analyzed two duopoly models for complements. From the first order necessary and the second order sufficient conditions, it was shown how the optimal inventory and pricing policies were derived. As a special case, the symmetric demand and cost were assumed and the closed form inventory and pricing policies were obtained at equilibrium. Comparing substitutes with complements numerically, we showed that the Bertrand-type competition was more efficient than the Cournot-type competition and the sellers tended to have higher EOQ for complements than that for substitutes.

Future Research

In this section, we proceed to describe our future research direction. Even though we have focused on inventory and investment policies as well as inventory and pricing policies in this dissertation, it would be of interest to study the effects of investment and pricing policies simultaneously in order to analyze inventory reduction.

Furthermore, we can extend our single product inventory model with the capital budget constraint to several products that have economic relations (i.e., substitutes and complements). Furthermore, it would be of interest to analyze various scenarios such as rework and/or salvage value of the defective items by considering not only investment policies but also pricing policies. By relaxing traditional EOQ assumptions, our models include incorporation of more sophisticated features such as shortages, delivery lags, and stochastic demand rates, etc. In addition, it is interesting to analyze inventory

and investment policies or inventory and pricing policies by employing more general relationships (i.e., nonlinear cases).

GENERAL REFERENCES

Arcelus, F. J. and Rowcroft, J. E. (1992). All-units quantity-freight discounts with disposals. *European Journal of Operational Research*, *57*, 77–88.

Banks, Jerry, and Fabrycky, W. J. (1987). *Procurement and inventory systems analysis.* Englewood, NJ: Prentice-Hall Inc.

Chen, Cheng-Kang (1995). *Market-based allocation mechanisms for lot-size decision makers and electric power utilities* (Ph.D. dissertation). Ames, Iowa: Iowa State University.

Cheng, T.C.E. (1991). EPQ with process capability and quality assurance considerations. *Journal of Operations Research Society*, *42*(8), 713–720.

Hariga, Moncer A. (1994). Economic analysis of dynamic inventory models with non-stationary costs and demand. *International Journal of Production Economics*, *36*, 255–266.

Hillier, Frederick S. and Lieberman, Gerald J. (1995). *Introduction to operations research*, sixth edition. New York, New York: McGraw-Hill Publishing Company.

Hong, Jae-Dong, Xu, S. H., and Hayya, J. C. (1993). Process quality improvement and setup reduction in dynamic lot-sizing. *International Journal of Production Research*, *33*(11), 2693–2708.

Johnson, Lynwood A. and Montgomery, Douglas C. (1974). *Operations research in production planning, scheduling, and inventory control.* New York, NY: John Wiley & Sons.

Ladany, S. and Sternlieb, A. (1974). The interaction of economic ordering quantities and marketing policies. *AIIE Transactions*, *6*(1), 35–40.

Lee, Hau L. and Rosenblatt, Meir J. (1985). Optimal inspection and ordering policies for products with imperfect quality. *IIE Transactions*, *17*(3), 284–289.

Liao, Ching-Jong and Shyu, Chih-Hsiung (1991). Stochastic inventory model with controllable lead time. *International Journal of Systems Science*, *22*(11), 2347–2354.

Nahmias, Steven (1989). *Production and operations analysis.* Homewood, IL: Irwin.

Porteus, Evan L. (1985). Investing in reduced setups in the EOQ model. *Management Science*, *31*(8), 998–1010.

Rosenberg, David. (1991). Optimal price-inventory decisions: profit vs. ROII. *IIE Transactions*, *23*(1), 17–22.

Schroeder, Roger G. and Krishnan, Ramakrishnan (1976). Return on investment as a criterion for inventory models. *Decision Sciences*, *7*(4), 697–704.

Silver, Edward A., and Peterson, Rein (1985). *Decision systems for inventory management and production planning*, second edition. New York, NY: John Wiley & Sons.

Varian, H. R. (1992). *Microeconomic Analysis*, New York, New York: W. W. Norton & Company.

Varian, H. R. (1992). *Microeconomic Analysis*, New York, New York: W. W. Norton & Company.

Voss, C.A. (1987). *International trends in manufacturing technology: Just-In-Time manufacture.* Linz, Austria: IFS Publications.

Whitin, T. M. (1955). Inventory control and price theory. *Management Science*, *2*, 61–68.

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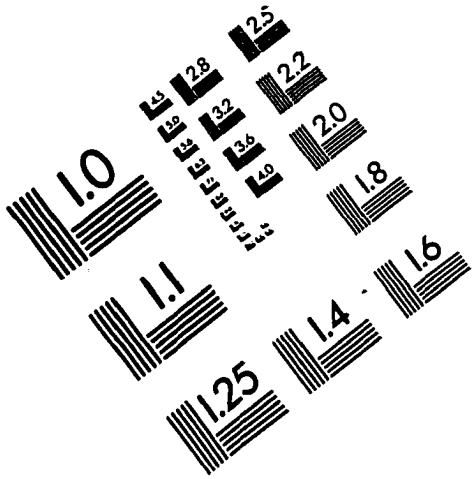
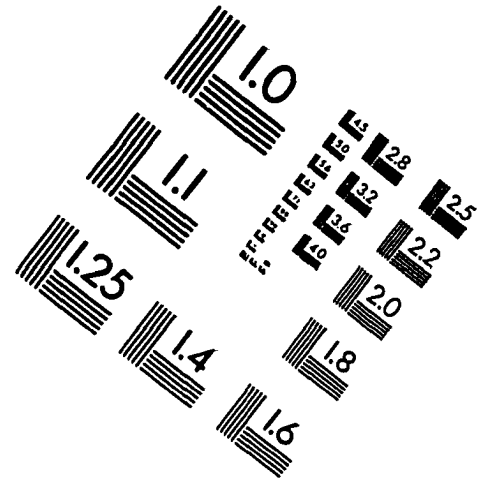
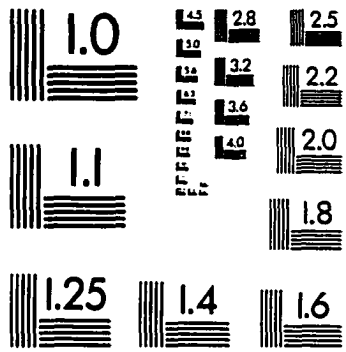
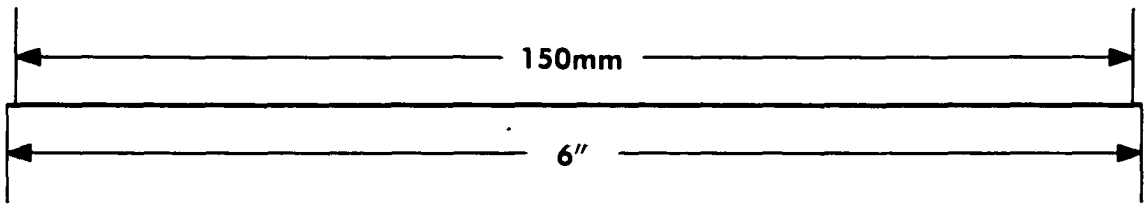
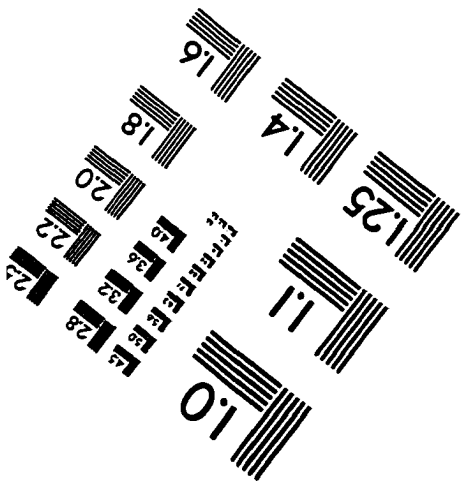
IMAGE EVALUATION TEST TARGET (QA-3)

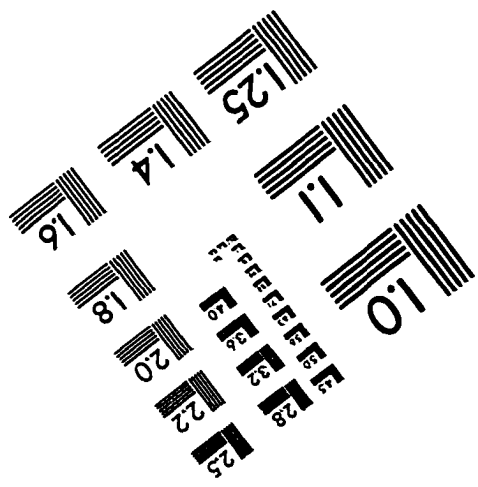












APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989